Fast Inductance Extraction of 3-D Structures with Non-constant Permeabilities

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ABSTRACT

In this paper we present a discretized integral formulation for calculating the frequency-dependent inductance and resistance for 3-D structures that contains permeable materials. The method uses a magnetic surface charge formulation, and we present analytic techiques for evaluating the required integrals. Computational results are presented and compared with analytic formulas to demonstrate the accuracy and versatility of the approach.

Keywords: Inductance, Magnetic, permeability.

INTRODUCTION

In order to generate large displacements in micromechanical devices, several new designs generate large forces using magnetic fields combined with high permeable materials [1]. To efficiently analyze the forces in these complicated 3-D structures with permeable materials, we have been working on approaches to extend the FastHenry 3-D inductance extraction program [2] to include non-constant permeability structures. The approach used is based on including fictitious magnetic surface charges [3]. This method avoids numerically calculating fields inside the high permeability materials, as these small fields are difficult to compute accurately due to cancellation errors.

In the next section we briefly describe the FastHenry program. In Section 3, we describe our integral formulation and show how the individual integrals can be evaluated efficiently. In Section 4 we present two computational experiments. In the first experiment we demonstrate the validity of our formulation and discretization by comparing our numerically computed results to analytically derived results for a simple structure. We then show results on a more complicated example which exhibits frequency dependent inductance, and variation of inductance with permeability. The paper ends with conclusions and acknowledgements.

FASTHENRY BACKGROUND

FastHenry[2] uses a standard filament discretization of an integral formulation of magnetoquasistatics[6]. The

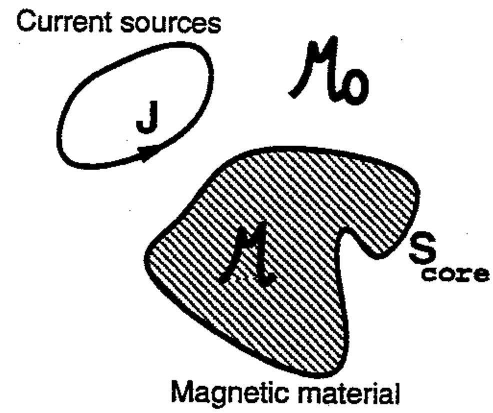

Figure 1: Current sources are outside the magnetic material.

integral equation is

$$\frac{J(r)}{\sigma} + \frac{jw\mu}{4\pi}\int_{V'}\frac{J(r')}{|r-r'|}dv' = -\nabla\Phi(r), \tag{1}$$

where Φ is referred to as the scalar potential, and V' is the volume of all conductors. The conductor current density, J, and the scalar potential can be computed by simultaneously solving (1) with the current conservation equation,

$$\nabla \cdot J = 0. \tag{2}$$

Discretization of (1) and (2) leads to a dense linear system of equations, and the FastHenry Program solves that system efficiently by combining a loop formulation with a multipole-accelerated iterative method[5,7].

MAGNETIC INTEGRAL FORMULATION

Coupled Integral Equation

We assumed that regions which contain magnetic material are separated from current carrying conductors, as shown in Figure 1, as it is the case in many magnetic problems.

In our method for handling permeable materials, we get two coupled integral equations. The first integral equation is a modification of (1),

$$\frac{J(r)}{\sigma}+\frac{j\omega\mu}{4\pi}\int_V^{\prime}\frac{J(r')}{|r-r'|}dv'+$$
$$\frac{j\omega\mu}{4\pi}\nabla_r\int_{S_m'}\frac{-\rho_m(r')}{|r-r'|}dS_m'=-\nabla\phi(r) \quad (3)$$

where J is the current density, ρ_m is the fictitious magnetic charge density on the permeable material surface, S_m is the surface of the permeable material, and ϕ is the scalar potential. Note that the second term and the third term of (3), divided by $j\omega\mu$, represent the vector potential due to the currents and the the vector potential due to fictitious magnetic charges, respectively.

The second integral equation is derived by applying the boundary condition of the continuity of the normal magnetic flux density in free space and in the magnetic material. For a point on the magnetic material interface the following equation is satisfied,

$$\rho_m(r)\frac{2\pi(\mu_r+1)}{(\mu_r-1)}=\nabla\times\int_v\frac{J(r')dv'\cdot n(r)}{|r-r'|}-$$
$$\int_s\rho_m(r')n(r)\cdot\nabla\frac{1}{|r-r'|}dS_{core}' \quad (4)$$

where n(r) is the unit vector normal to the magnetic material surface calculated at point r. Note that the second term of (4) represents the normal magnetic field due to currents, and the third term represents the normal magnetic field due to fictitious magnetic charges.

Discretization

Equations (3) and (4) are discretized by dividing the conductors into filaments over which the current is assumed constant and by dividing the permeable material surface into panels over which the magnetic charge is assumed constant. Then, the filaments are combined into loops. By using these loop and panel basis functions to represent the currents and charges, we convert (3) and (4) to a system that can be solved numerically.

For each current loop, the following equation is formulated

$$V_{loop_{Mi}}=R_{loop_{Mi}}I_{Mi}+$$
$$j\omega\sum_{Mj}\frac{\mu I_{Mj}}{4\pi a_i a_j}\int_V\int_V^{\prime}\frac{u_{Mi}\cdot u_{Mj}}{|r-r'|}dv_i dv_j'-$$
$$\frac{j\omega\mu}{4\pi}\sum_{mj}q_{mj}\int_{S_{loop}}\nabla_r\int\frac{1}{|r-r'|}dS_{core}'\Delta_{mj}dS_{loop_{Mi}} \quad (5)$$

where $R_{loop_{Mi}}$, I_{Mi}, and $V_{loop_{Mi}}$ are the resistance of, the current in, and voltage across the i^{th} current loop respectively. q_{m_j} is the fictitious magnetic charge density on the j^{th}panel on the magnetic material surface. u_{Mi} is the unit vector in the i^{th} loop direction.

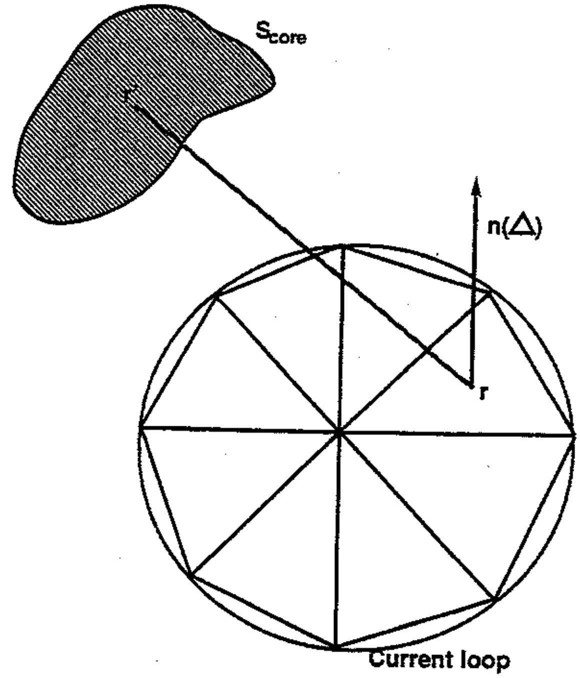

Figure 2: The surface of the current loop are divided into triangles in calculating the integral L_ρ.

The discretized boundary condition equation is

$$q_{mi}\frac{2\pi(\mu_r+1)}{(\mu_r-1)}=\sum_{Mj}I_{Mj}\int_V\nabla\times\frac{u_{Mj}dv'\cdot n_{mi}}{|r-r'|}$$
$$-\sum_{mj}q_{mj}\int_S n_{mi}\cdot\nabla\frac{1}{|r-r'|}dS_{\Delta_{mj}}' \quad (6)$$

where μ_r is the relative permeability of the magnetic material, and n_p is the unit vector normal to the magnetic material surface calculated at point p.

Equations (5,6) can be summarized in matrix form as

$$\begin{bmatrix} R(\omega)+j\omega L_J(\omega) & j\omega L_\rho(\omega) \\ Hn_J & (Hn_\rho - I)\end{bmatrix}\begin{bmatrix}\bar{I}_M \\ \bar{q}_m\end{bmatrix}=\begin{bmatrix}V\\0\end{bmatrix} \quad (7)$$

Calculating the integrals

Consider the term corresponding to L_ρ in (7), which comes from integrating 1/r over the surface of a current loop and evaluating the result at point at the permeable material surface. Since the shape of the surface of the current loop is arbitrary, we chose it to be a tented surface, as shown in Figure 2. This tented surface is composed of triangles, and so L_ρ is given by:

$$L_\rho=\frac{-\mu_0}{4\pi}\sum_\Delta\int\rho_m(r')dS_{core}'\nabla_r\frac{1}{|r-r'|}\cdot n(\Delta)dS_\Delta \quad (8)$$

where $n(\Delta)$ is the normal to triangle on the current loop surface. The integral $\int\rho_m(r')dS_{core}'\nabla_r\frac{1}{|r-r'|}\cdot n(\Delta)dS_\Delta$ is the potential due to a dipole charge distribution on a

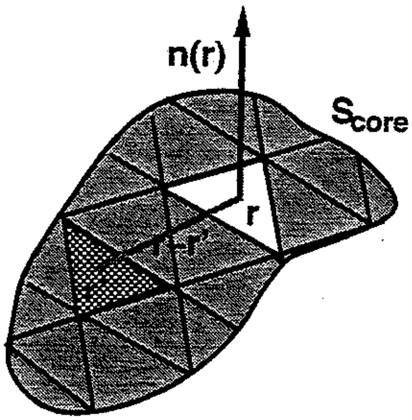

Figure 3: The surface of the permeable material is divided into triangles in calculating the integral Hn_ρ.

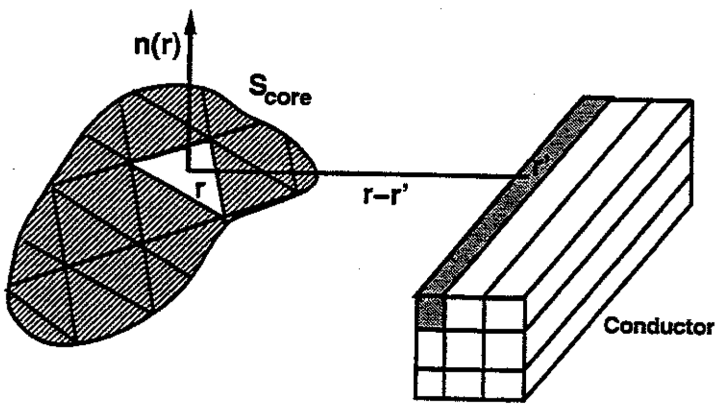

Figure 4: The conductor are discretized into filaments in calculating the integral Hn_J.

triangles. This potential can be evaluated analytically [8,9].

Now consider the Hn_ρ term in equation (7). This term is equivalent to calculating the normal magnetic field at a point due to a magnetic charge distribution. Having discretized the permeable material surface into triangular panels, as shown in Figure 3, made it possible to calculate Hn_ρ analytically[8].

The last term that to be considered is Hn_J. The conductors are discretized into filaments, as shown in Figure 4. Then by transforming each filaments to the panel coordinates, an analytical formula can be derived and used for Hn_J.

As described above, each of the terms in equation (7) can be evaluated analytically.

RESULTS

To demonstrate the correctness of this formulation, consider the example of a current loop over an infinite thickness magnetic substrate, shown in Figure 5. Assuming low frequency, the current distribution will be uniform across the current loop. Thus, the inductance

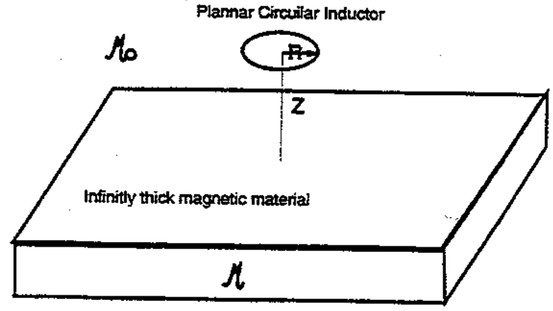

Figure 5: Planar circular current loop of radius R, and hight Z above an infinitely thick magnetic material of permeability μ.

determined by solving equation (7) can be written as $L = L_f + L_M$ where L_f is the self inductance of the loop if there were no magnetic substrate, and L_M is the extra inductance due to the presence of the magnetic material. L_f is equal to the second term in Equation 1, divided by jw. The term L_M can be computed using the fictitious charge method, but in this simple example L_M can also be determined by the method of images. In Table 1, we compare L_M computed by our fictitious charge numerical method with that computed using the less general method of images and the tables in [4]. As Table 1 demonstrates, the two methods agree.

R	Z	Numerical L_M	L_M by image	Error
5	1	$6.7076XnH$	$6.75XnH$	0.6%
4	4	$0.5652XnH$	$0.5674XnH$	0.4%
2	1	$0.9817XnH$	$0.9882XnH$	0.7%
1	2	$0.0258XnH$	$0.0260XnH$	0.3%
1	3	$8.4099XpH$	$8.438XpH$	0.8%

Table 1: Comparison between L_M calculated using our method and the one using the image principle and tables in [4]. Dimensions of R and Z are in mm. $X = \frac{\mu_r - 1}{\mu_r + 1}$.

Generally, the conductor current distribution is not known a priori, and must be computed by solving the coupled integral equation. As an example, consider a current loop of square cross section surrounding a spherical magnetic material, as shown in Figure 6.

Figure 7 shows the frequency response of the total inductance of the example in Figure 6. Note the high frequency inductance is higher than the low frequency one, due to the skin effect. The frequency at which the inductance start to drop is mainly determined by the size of the cross section of the conductor.

Figure 8 shows that for the example in Figure 6, the inductance increases as the permeability of the magnetic material increases, up to a limiting value. The inductance for the $\mu = 1$ case matches exactly the self inductance of the loop calculated by FastHenry.

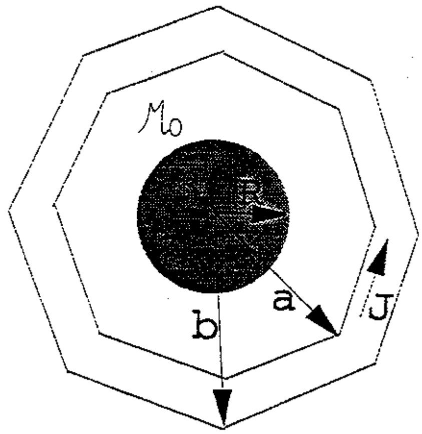

Figure 6: current loop of square cross section surrounding a spherical magnetic material. R=1mm, a=1.75mm, b=2.25mm.

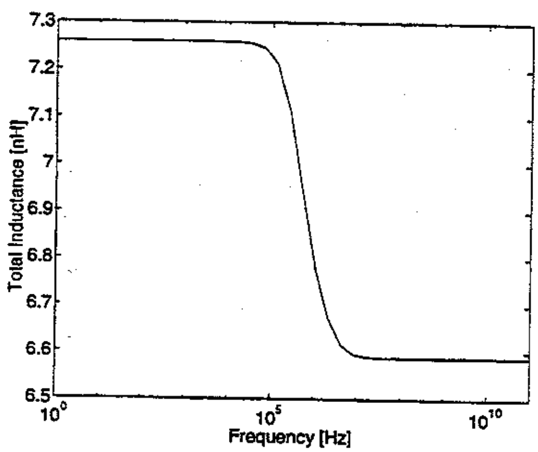

Figure 7: Total inductance frequency response of the structure in Figure 7

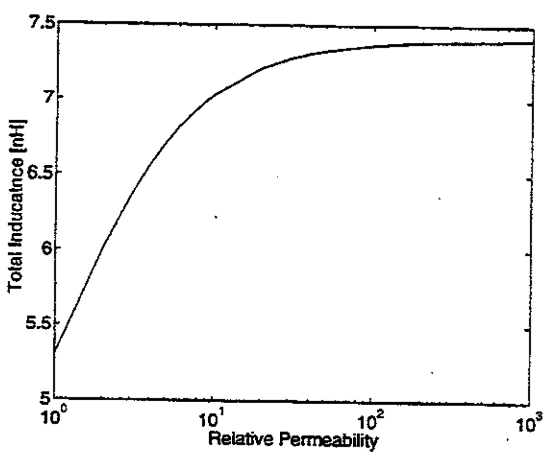

Figure 8: Total inductance variation with the permeability of the magnetic material for the structure in Figure 6

CONCLUSIONS AND ACKNOWLEDGEMENTS

An integral formulation for calculating 3-D inductance for structures that contains permeable materials has been developed. The method avoids numerical cancellation errors that occur when calculating fields inside highly permeable materials. We showed that for a simple discretization technique, the integrals in the formulation can be computed analytically. Computational results were presented and compared with analytic formulas to demonstrate the accuracy and versatility of the approach. We expect to be able to combine our permeable material approach with the multipole-accelerated iterative method used in FastHenry so that very complicated 3-D structures with permeable materials can be analyzed quickly.

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