Automatic Generation of Small-Signal Dynamic Macromodels from 3-D Simulation

Deepak Ramaswamy and Jacob White

Massachusetts Institute of Technology
36-880, 50 Vassar Street, Cambridge, MA 02139
deepak,white@rle-vlsi.mit.edu, Ph: 617-2538169

ABSTRACT

Recent algorithmic developments have greatly accelerated 3-D simulation of micromachined devices, but simulation and optimization of systems which use those devices require much more easily evaluated, yet still accurate, macromodels. In this paper we focus on generating dynamically accurate small-signal macromodels, useful in many signal processing and feedback control applications. Results are presented demonstrating that dynamically accurate macromodels for the small-signal behavior of a cantilever beam and a micromirror can be automatically generated directly from 3-D simulation.

Keywords: MEMS, Model, Reduction, Arnoldi, Electromechanical

1 INTRODUCTION

The micro-mirror (Figure 1) and a simple cantilever beam are examples of coupled micro-electro-mechanical (MEMS) systems. These systems are coupled in the sense that the mechanical deformation is determined by the electrostatic forces and the electrostatic forces in turn depend on the deformation. It is possible to analyze such structures using recently developed accelerated methods ([1], [2]), but these methods are still too slow to be used in system-level simulation and optimization. Some form of macromodeling, or model-reduction, is required.

There are many approaches to model order reduction (i.e. [3], [4]), and herein we describe an approach based on adapting the techniques in [3], [5]. These previous approaches exploited the fact that system matrices were explicitly available, and that is not the case when using accelerated 3-D solvers. In addition, much more accurate models can be developed by incorporating *both* the linear and quadratic dependencies of the electrostatic forces on the applied voltage.

The basic idea behind many model order reduction techniques is to write down the algebraic relation between the input and output in the frequency domain and then somehow approximate the transfer function with a much lower order system. Most electromechanical systems with no damping can be described by the

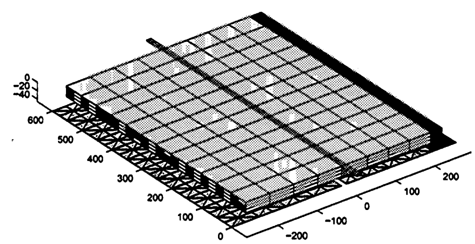

Figure 1: Micromirror geometry. Note the pair of thin drive electrodes beneath the mirror.

O.D.E.

$$\begin{aligned} &M\ddot{u} + Ku = bv \\ &y = c^T u \\ &Y(s) = \underbrace{c^T(s^2M+K)^{-1}b}_{H(s)}\ U(s) \end{aligned} \tag{1}$$

where M is the mass matrix, K the stiffness matrix, b the input force direction and v is the input.

The transfer function can be expanded as a power series about $s = 0$, in which case the coefficients of the power series are

$$c^T(K^{-1}M)^{i-1}K^{-1}b,\ i = 0\ldots\infty.$$

A transformation matrix V can be defined as containing the Krylov subspace

$$V \supset <\hat{K}^{-1}\hat{b}, (\hat{K}^{-1}\hat{M})\hat{K}^{-1}\hat{b} \ldots (\hat{K}^{-1}\hat{M})^{i-1}\hat{K}^{-1}\hat{b}> \tag{2}$$

and then a variable transformation can be defined as

$$\begin{aligned} &u = Vz,\ \tilde{M} = V^TMV,\ \tilde{K} = V^TKV, \\ &\tilde{b} = Vb,\ \tilde{c} = Vc \end{aligned} \tag{3}$$

Using the variable transformation generates a reduced O.D.E.

$$\begin{aligned} &\tilde{M}\ddot{z} + \tilde{K}z = \tilde{b}v \\ &y = \tilde{c}^T z \end{aligned} \tag{4}$$

If V is defined as above, the power series expansion of the reduced system transfer function is guaranteed to match the first i terms of the power series expansion of the original system [3].

If the system has damping, then the O.D.E. becomes

$$\begin{array}{l} M\ddot{u} + D\dot{u} + Ku = bv \\ y = c^T u \\ Y(s) = \underbrace{c^T(s^2M + sD + K)^{-1}b}_{H(s)}\ U(s) \end{array} \tag{5}$$

where D is the damping matrix.

To generate a power series expansion for the transfer function, consider converting the system to first order,

$$\begin{array}{l} \hat{M} = \begin{bmatrix} I & \\ & M \end{bmatrix},\ \hat{K} = \begin{bmatrix} 0 & -I \\ K & D \end{bmatrix},\ \hat{b} = \begin{bmatrix} 0 \\ b \end{bmatrix} \\ \hat{M}\dot{z} + \hat{K}z = \hat{b}v \end{array} \tag{6}$$

If, as before, we define V as

$$V \supset < \hat{K}^{-1}\hat{b}, (\hat{K}^{-1}\hat{M})\hat{K}^{-1}\hat{b} \dots (\hat{K}^{-1}\hat{M})^{i-1}\hat{K}^{-1}\hat{b} >$$

then we can obtain reduced matrices, though they are not guaranteed to be stable. Instead [3] has shown if we take only the top half,

$$\hat{V}^1,\ \hat{V} = \begin{bmatrix} \hat{V}^1 \\ \hat{V}^2 \end{bmatrix}$$

the resulting reduced model matches exactly i power series terms The intuition is that if $x = V_1 z$, then the time derivative should be $\dot{x} = V_1\dot{z}$ and not $\dot{x} = V_2\dot{z}$. Note also that the columns in V_1 span more than the space spanned by V_2.

The reduced matrices are defined as

$$\begin{array}{l} \tilde{u} = \hat{V}^1 u,\ \tilde{M} = (\hat{V}^1)^T M \hat{V}^1,\ \tilde{K} = (\hat{V}^1)^T K \hat{V}^1, \\ \tilde{D} = (\hat{V}^1)^T D \hat{V}^1,\ \tilde{b} = \hat{V}^1 b,\ \tilde{c} = \hat{V}^1 c \end{array} \tag{7}$$

and the reduced O.D.E. is

$$\begin{array}{l} \tilde{M}\ddot{\tilde{u}} + \tilde{D}\dot{\tilde{u}} + \tilde{K}\tilde{u} = \tilde{b}v \\ y = \tilde{c}^T\tilde{u} \end{array} \tag{8}$$

Note that even though "K^{-1}" appears twice in

$$\hat{K}^{-1} = \begin{bmatrix} K^{-1}D & K^{-1} \\ -I & 0 \end{bmatrix}$$

K^{-1} needs to applied only once to compute "$\hat{K}^{-1}\times$ a vector". One can also alternatively expand around $s = \infty$. In this case the reduced model will match (1) near $t = 0$ as opposed to the $s = 0$ reduced model described here which matches the steady state of (1).

2 COUPLED DOMAIN REDUCTION

The equations for a coupled domain system (without damping) are

$$\begin{array}{l} M\ddot{x} + F(x) = P(x, q) \\ A(x)^{-1}q = \phi_0 + \Delta\phi v \end{array} \tag{9}$$

where x is the state space, F the force due to internal stresses, P the external surface force which depends on the charge q, A is the potential coefficient matrix, ϕ_0 is the bias potential vector which can undergo a perturbation v in the direction $\Delta\phi$. $\Delta\phi$ consists of ones correspoding to surfaces on which ϕ_0 is perturbed and zeroes otherwise. Let us the assume that the original fully nonlinear model is completely elastic and that we would like to generate a completely elastic reduced model. To apply the aforementioned model reduction technique, the strategy here will be to linearize the above equations about the bias point (x_0, ϕ_0). Specifically,

$$\begin{array}{rcl} M\ddot{u} + \underbrace{(\frac{\partial F}{\partial x} - \frac{\partial P}{\partial x} - \frac{\partial P}{\partial q}\frac{\partial q}{\partial x})}_{K} u & = & \underbrace{\frac{\partial P}{\partial q}\frac{\partial q}{\partial v}}_{RHS_1} v + \\ & & \underbrace{\frac{1}{2}\frac{\partial}{\partial v}\left[\frac{\partial P}{\partial q}\frac{\partial q}{\partial v}\right]}_{RHS_2} v^2 \end{array} \tag{10}$$

Observe that the right hand side is not strictly a linearization. The quadratic term is explicitly included as the electrostatic force is related to the square of the applied voltage.

At the bias point ϕ_0 the charge q_0 is given as

$$q_0 = A(x_0)\phi_0$$

Perturbing ϕ_0 by v, the change in q^2 is given by

$$\begin{array}{l} q_i^2 = \phi_0^T A(i,:)^T A(i,:)\phi_0 \\ \Delta q_i^2 = 2\phi_0^T A(i,:)^T A(i,:)\Delta\phi\, v + \Delta\phi^T A(i,:)^T A(i,:)\Delta\phi\, v^2 \end{array}$$

Since the force depends linearly on q_i^2, if the bias ϕ_0 is 0, a linearization will result in zero force and the quadratic term must be included. As a result we can expect that if the input is a sine wave of frequency ω the response has frequencies ω and 2ω. Note the computation is straightforward. $A(i,:)\phi_0$ is simply $q_0(i)$ and $A(i,:)\Delta\phi$ is i^{th} coordinate of the charge computed with $\Delta\phi$ as the voltage. Therefore computing the right hand side force term just involves an extra electrostatic black box call over the equilibrium charge calculation. Also note that prior to model order reduction we must have already explicitly computed $\frac{\partial P}{\partial q}$ (which is needed for solution of the outer Newton loop) at the equilibrium point. For convenience we now calculate "$\frac{\partial P}{\partial q^2}$". Also while $\frac{\partial F}{\partial x} - \frac{\partial P}{\partial x}$ is known explicitly, $\frac{\partial q}{\partial x}$ is computed using finite differences as in [1].

Calculating the Krylov subspace corresponding to the above equation can be very slow as the inverse of the K matrix has to be applied using an inner iterative procedure. This inner iteration converges slowly because of the wide range of eigenvalues of the stiffness matrix. Therefore since $(\frac{\partial F}{\partial x} - \frac{\partial P}{\partial x})$ is already known explicitly we factor it and use it as a preconditioner for finding

K^{-1}. As a consequence the number of iterations rarely exceeds four. Although V is orthogonalized while calculating the basis (and this also avoids ill-conditioning of the reduced matrices), in the damping case V_1 is not orthonormal and is therefore orthogonalized.

Also note that with a nonsymmetric K, there is no known way to diagonalize both M and K simultaneously. Since we have two right sides we can generate two sets of matrices

$(\tilde{M}_1, \tilde{D}_1, \tilde{K}_1, \tilde{b}_1, \tilde{c}_1, u_1)$, $(\tilde{M}_2, \tilde{D}_2, \tilde{K}_2, \tilde{b}_2, \tilde{c}_2, u_2)$

i.e. one for each right hand side and the actual solution is $u = u_1 + u_2$ (by linearity).

3 RESULTS

Most MEMS devices are not packed in vaccum and so we can expect to see air damping. But we will not incorporate air damping here. Instead we introduce a fictitious positive definite air damping matrix D to test our model reduction. First we reduce a "plain" linearized (i.e. no quadratic term of voltage) cantilever beam to a 15^{th} order model and compare it with the full simulation by taking it to steady state shown in Figure 3. The steady state error is large. For a lightly damped model we compare the transient responses of the fully nonlinear model, the 15^{th} reduced models of the "plain" linearized and linearized systems in Figure 4. The reduced model of the linearized system matches very well with the full model.

In the micromirror in Figure 1, the device input is a differential voltage applied to a pair of plates beneath the mirror, and the output is the micromirror's angular deflection. As shown in Figure 2 the quasistatic simulation results are in close agreement with experimental data [6]. The nearly 7000 degrees of freedom mirror is reduced into two 15^{th} order reduced models (one each for the linear and quadratic right hand sides (10)) without any damping and we see again that the fully elastic reduced model simulation matches well with the simulation of the fullmodel (rigid/elastic) [1].

However if the number of mechanical degrees of freedom is large as in the case of the micromirror the reduction process becomes expensive because of the cost of finding a static solution first/cost of factoring. In such cases it is possible to derive a reduced rigid/elastic model directly from the full rigid/elastic model.

4 Conclusion

We have successfully demonstrated a fully automatic technique to take a partly implicit system and reduce it to a much smaller explicit system that accurately captures the small signal behaviour of the original system.

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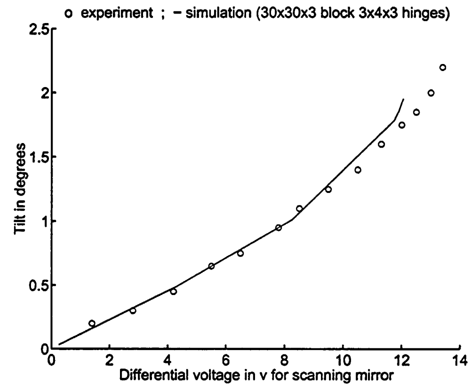

Figure 2: Micromirror displacement versus voltage.

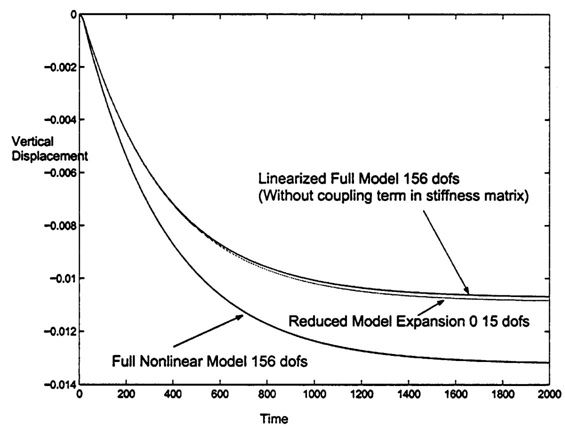

Figure 3: Cantilever beam voltage step responses (heavily damped case) using numerical simulation and a generated linear-only macromodel.

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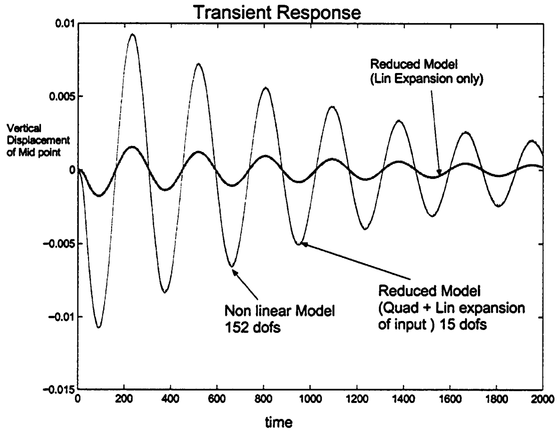

Figure 4: Cantilever beam voltage transient responses (lightly damped case) using numerical simulation and two generated macromodels.

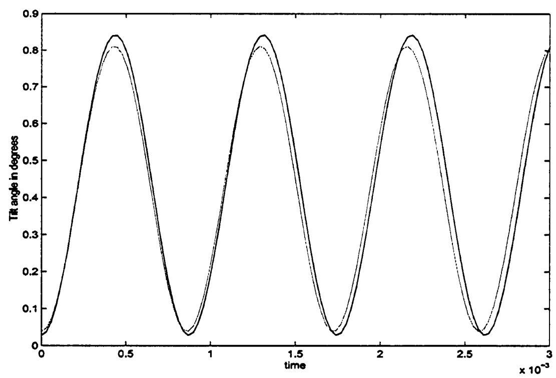

Figure 5: Comparing micromirror differential voltage step responses computing using numerical simulation and 15th order macromodels.

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