FastAero++

A pFFT Accelerated Advanced Panel Method

A pFFT Accelerated High Order Panel Method

By: David Willis

S.M. Thesis Research

Thanks: Prof. White, Prof. Peraire, Prof. Drela, SMA-MIT Alliance

Talk Outline

- **Introduction**
  - Panel Methods, History, Usage
  - Physics and Fluid Dynamics Involved
- **Integral Methods**
  - Introduction to the Integral formulation
  - The BIE('s) used in this work
- **Discretizing the BIE**
  - Basics of Discretization
  - Constant, & Linear Panel Integrals
  - Galerkin vs. Collocation
- **Acceleration of the Method**
  - Introduction to iterative methods
  - Introduction to pFFT
  - A general Linear pFFT implementation
- **Results, Conclusions and Recommendations**

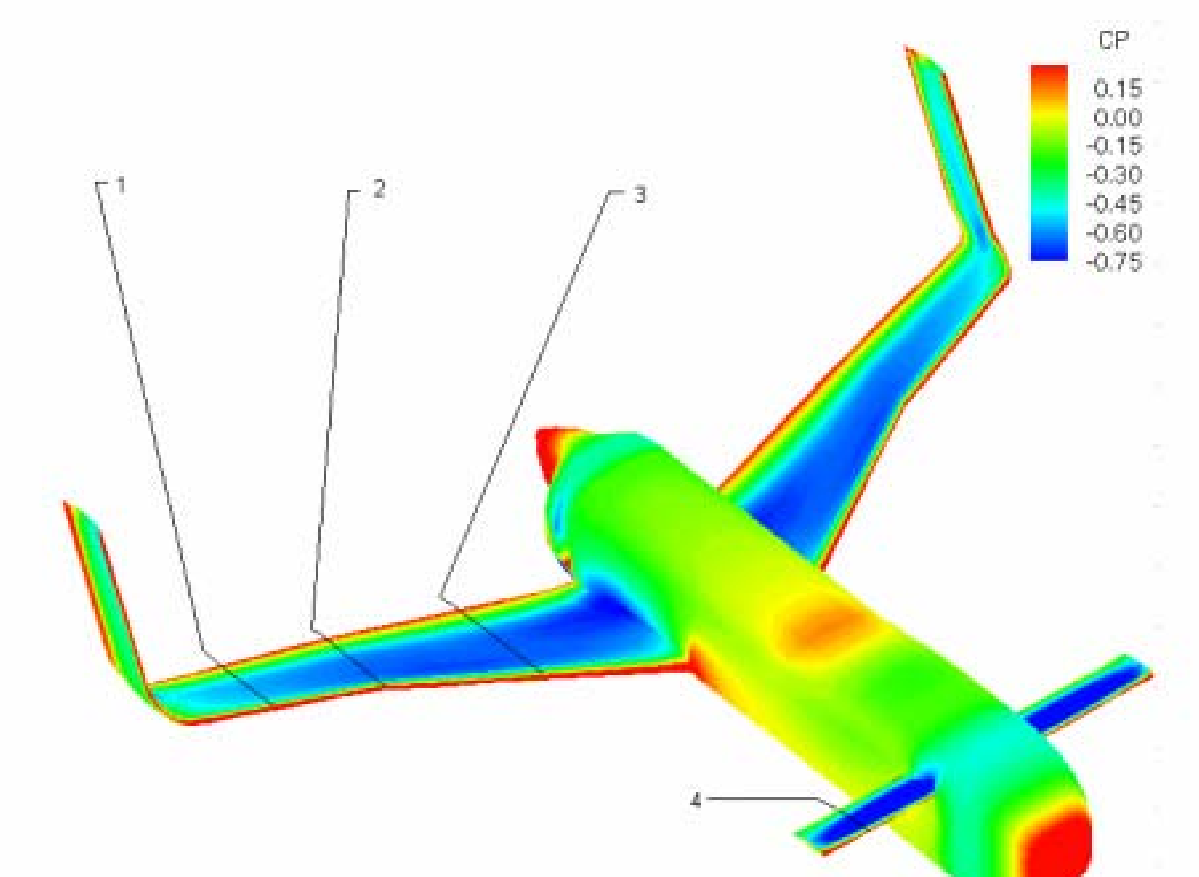
Introduction

Motivation For This Work

- Solve fluid flow around 3D bodies
  - Been done extensively, since c.1960's
- Why revisit?
  - Reduce solution time (significantly)
    - Problems requiring multiple solutions:
    - Stability and control
    - Optimization problems
    - B.L. Coupling Applications
    - Lot's of aerodynamic applications
- Primarily a design code
  - Fast, robust, accurate, non-separated flow
  - Not a Navier-Stokes solver!

Possible Applications

Carleton University Hammerhead UAV Project, 2000, David Willis



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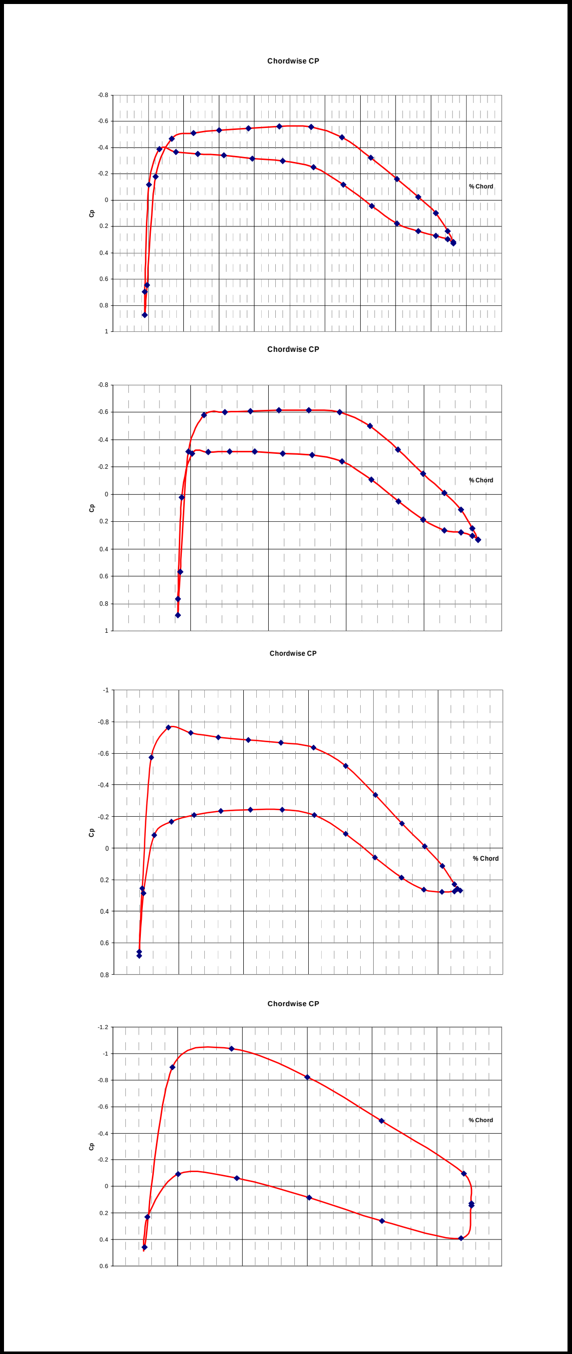
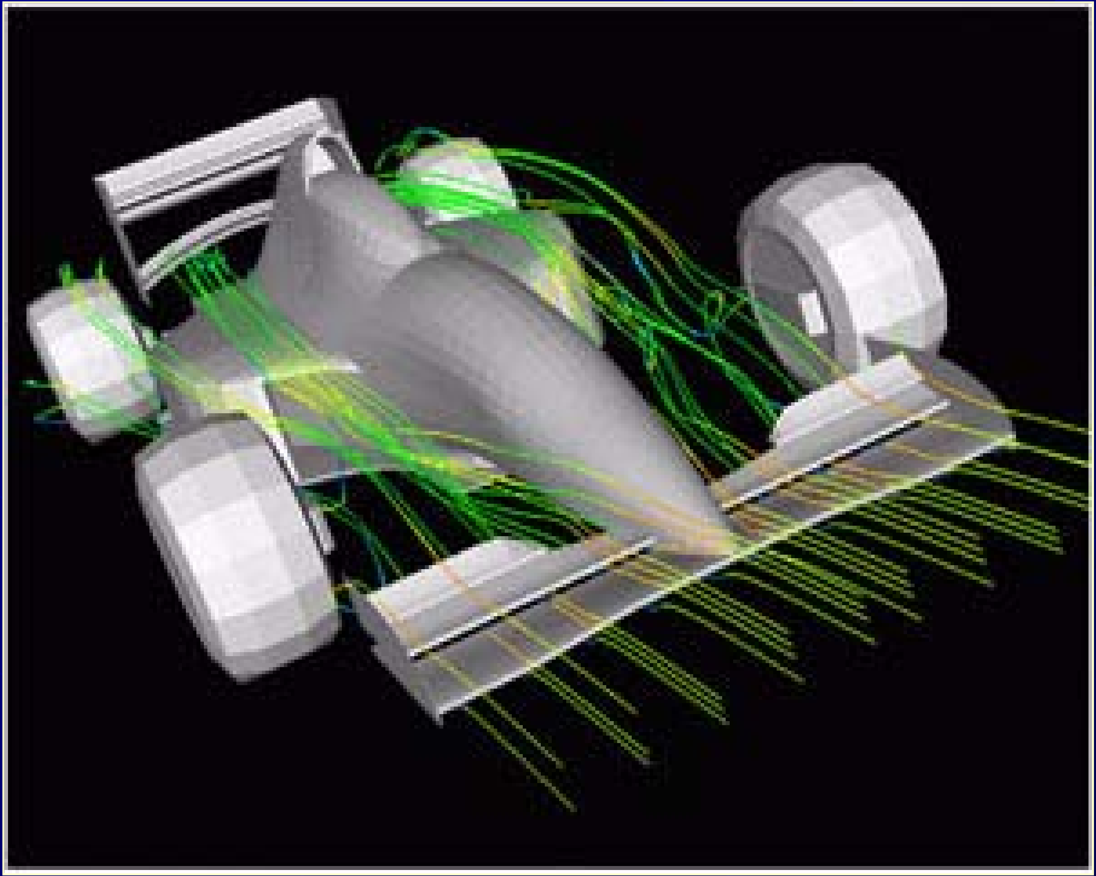


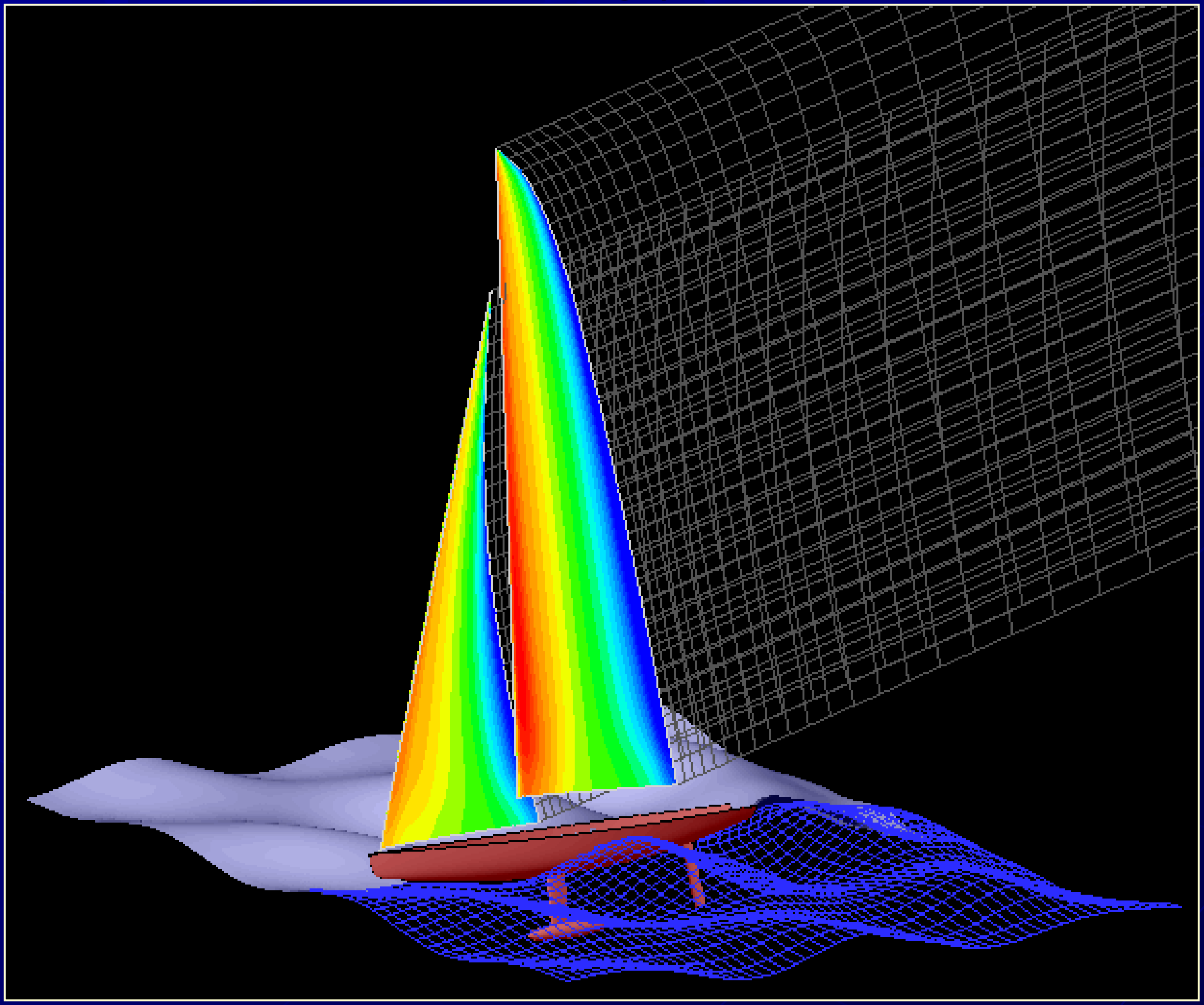
Other possible applications

- Automobile Aerodynamics : F-1 & Indy
  (http://www.flowsol.co.uk/index.html)



Other possible applications

- High Performance Sailing eg. IACC (http://oe.mit.edu/flowlab/websail.gif)



How Do We Model This?

- Start with the conservation of momentum and conservation of mass
  - Result is
    - Navier Stokes
    - Continuity
- Complicated Non-Linear Mess

ASSUMPTIONS!!!

Assumptions About the Flow

- Incompressible
- Steady
- Inviscid
- Irrotational

The resulting simplified equation is…

Potential Flow Equation

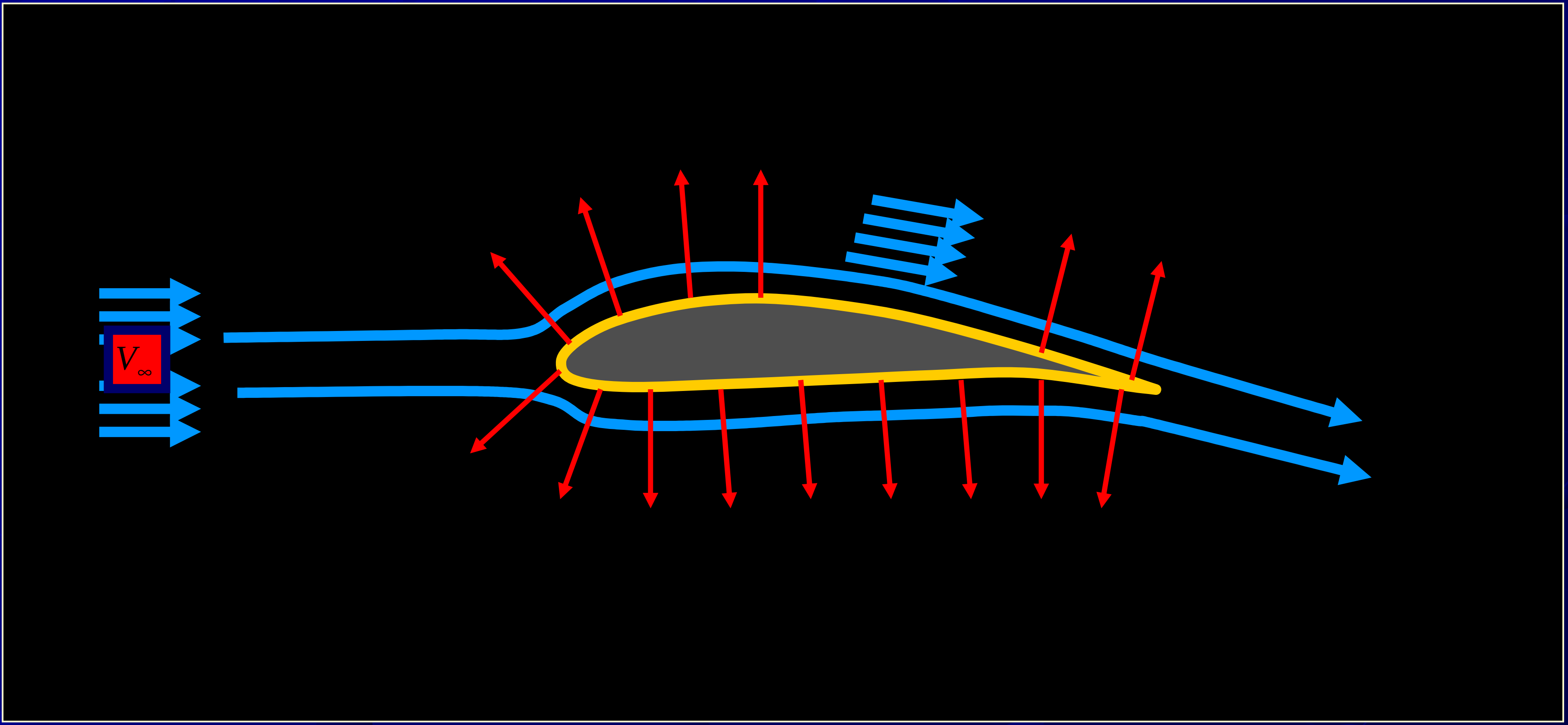
- Result Perturbation Potential on exterior domain:

$$\nabla^2 \varphi = 0$$

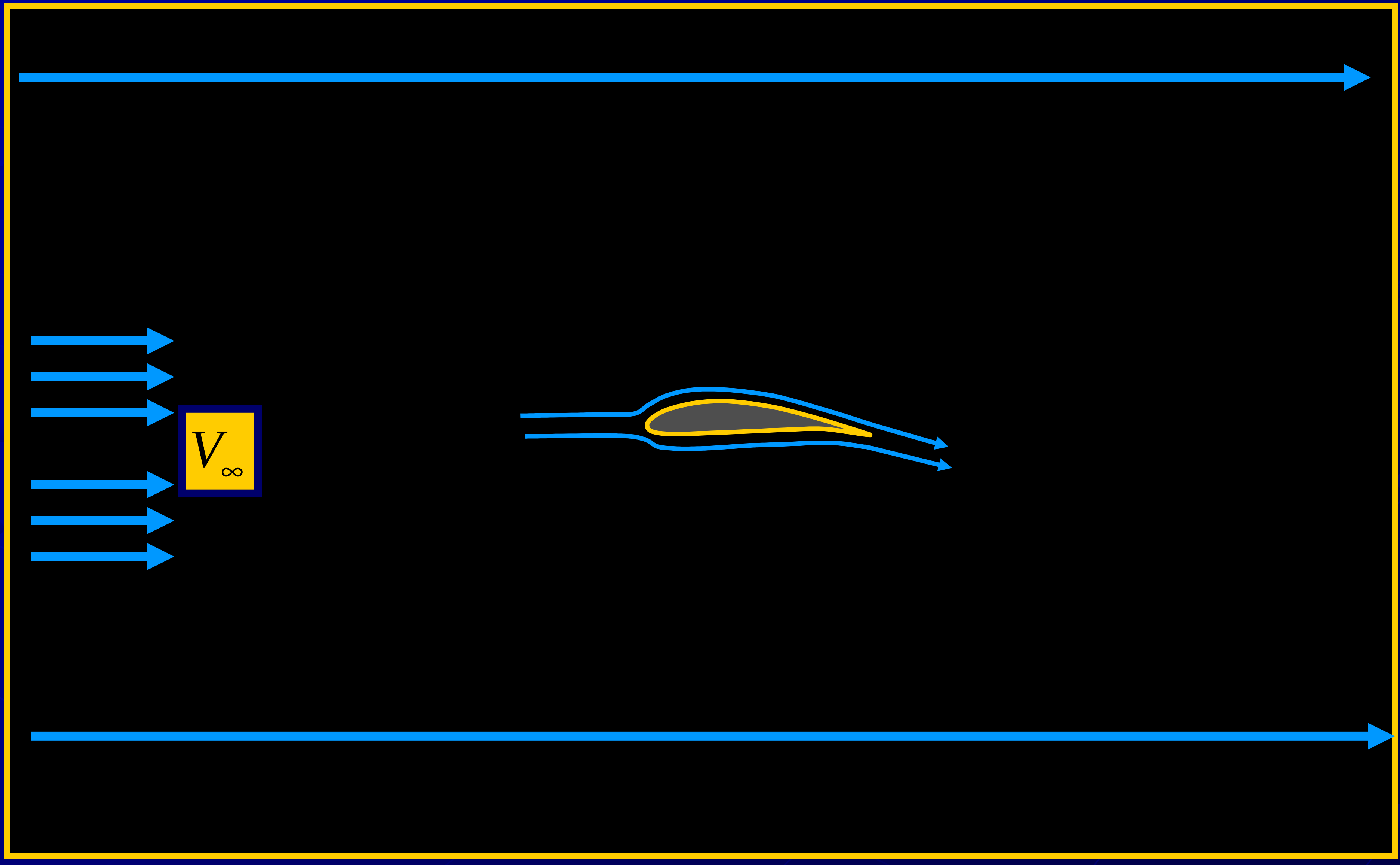
Laplacian operates on the “Velocity Potential”

Bernoulli relates Pressure & Velocity!

Body Boundary Conditions



Farfield Conditions



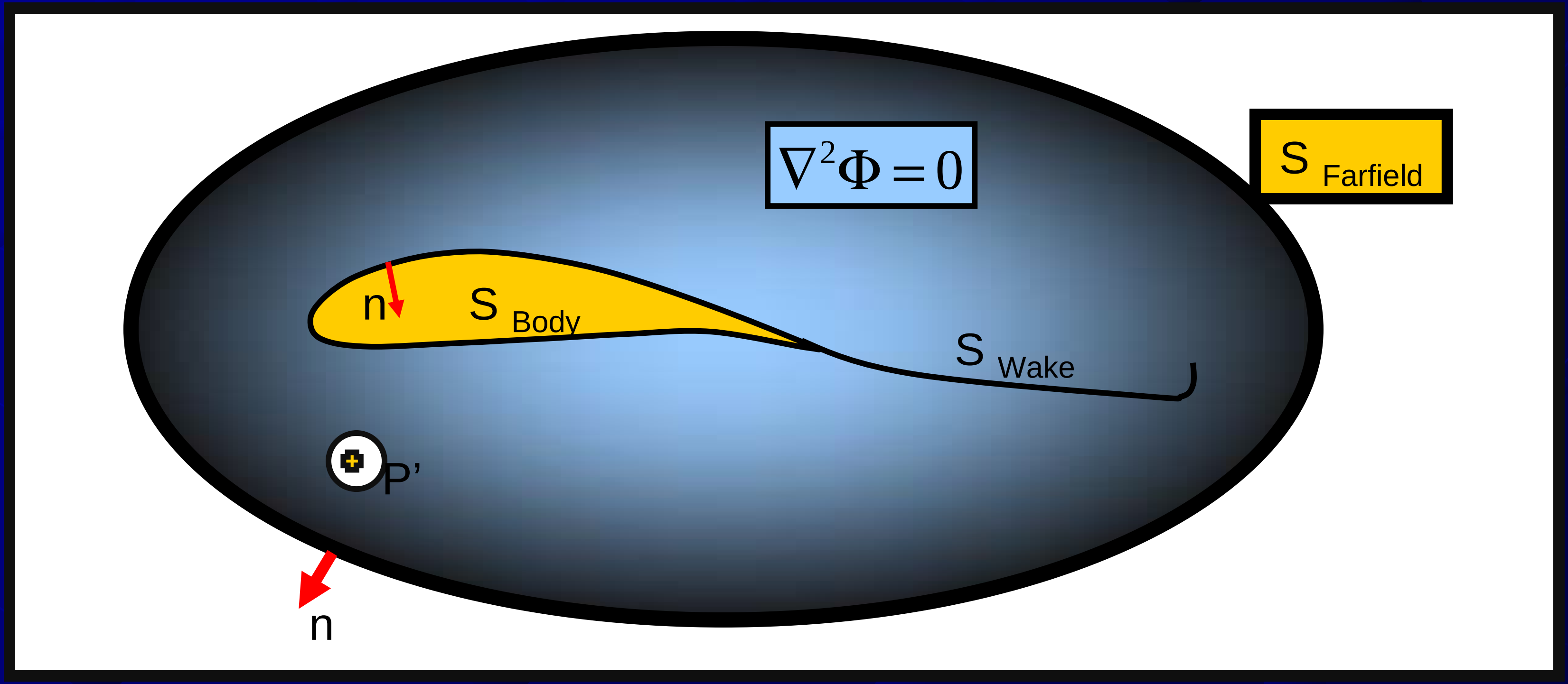
Boundary Integral Equations

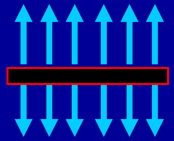
Introduction To Integral Equations

- Mathematical Appraoch:
  - Find Fundamental solution (Kernel)
  - Use Green's 3rd Identity
    - Transform from domain to boundary Integral
  - Use boundary condition data
- Aerodynamic Singularity Approach:
  - Source, doublet, and vortex singularities
    - Smear them on the boundary surface

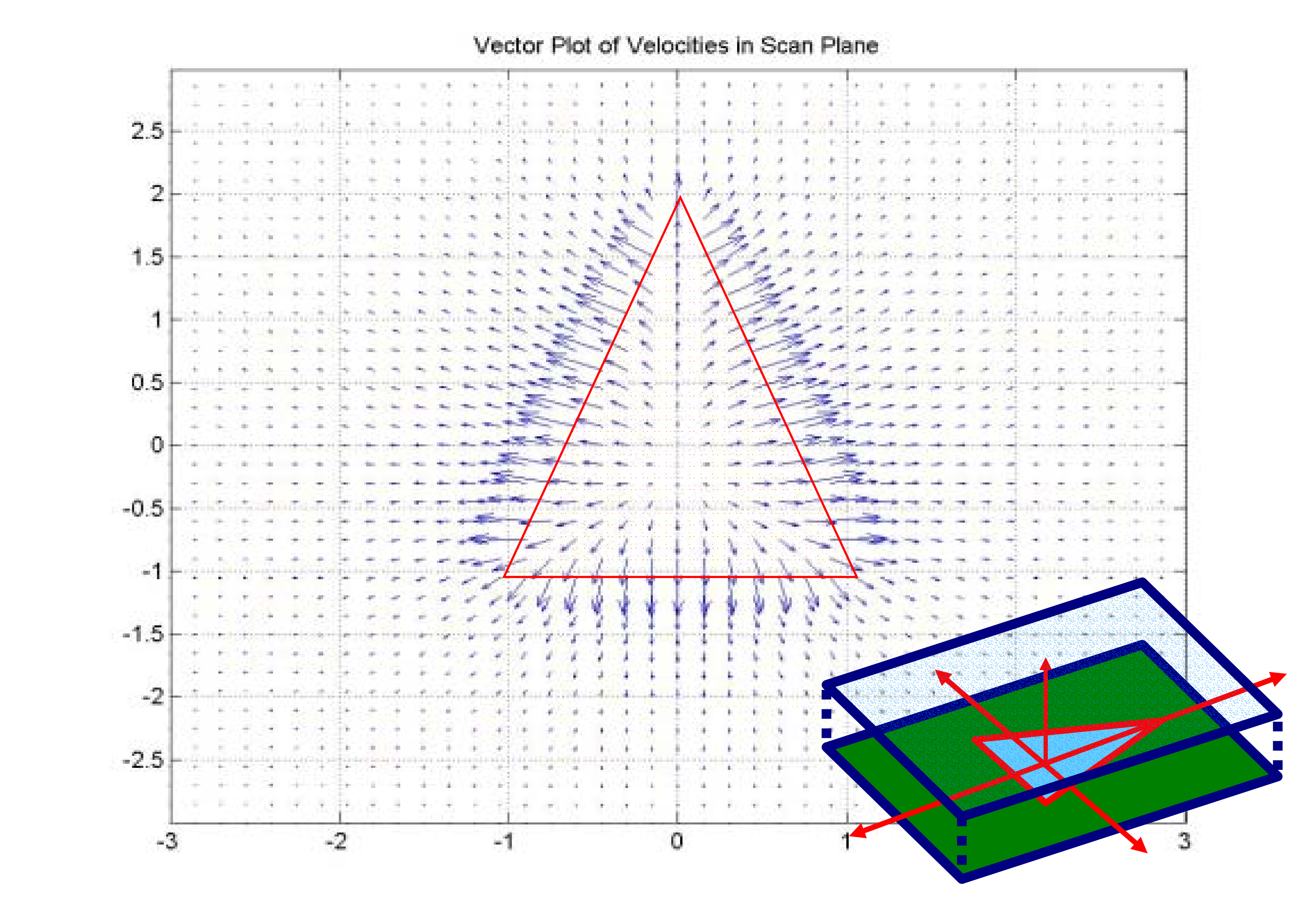
The 1/|r-r'| Integral Equation

$$\phi(\vec{x}') = \frac{1}{4\pi}\int\int_{S_B}\left(\frac{\partial\phi}{\partial n}\frac{1}{\|\vec{x}-\vec{x}'\|}\right)dS_B - \frac{1}{4\pi}\int\int_{S_{B+W}}\left(\phi\frac{\partial}{\partial n}\left(\frac{1}{\|\vec{x}-\vec{x}'\|}\right)\right)dS_{B+W}$$

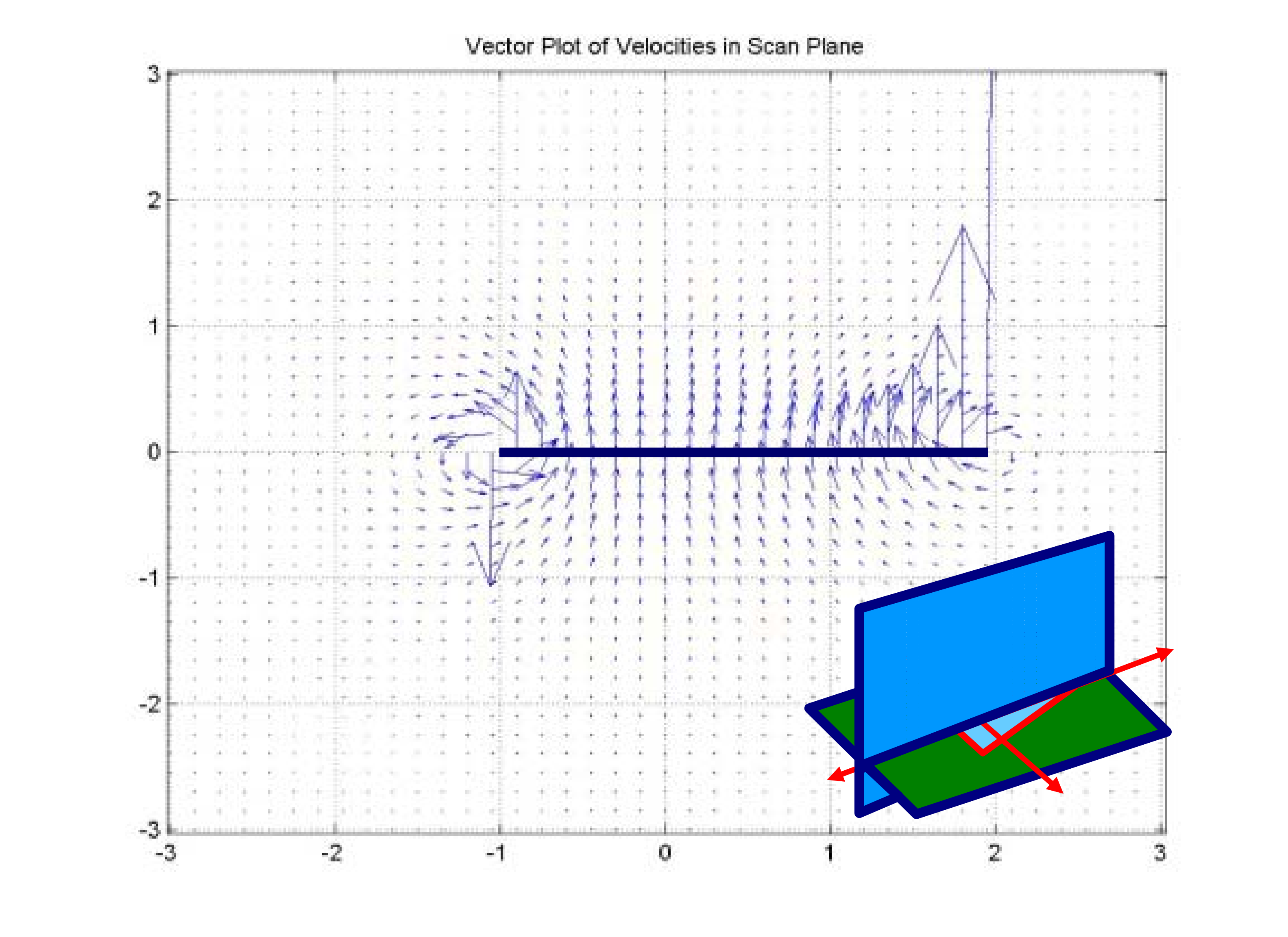




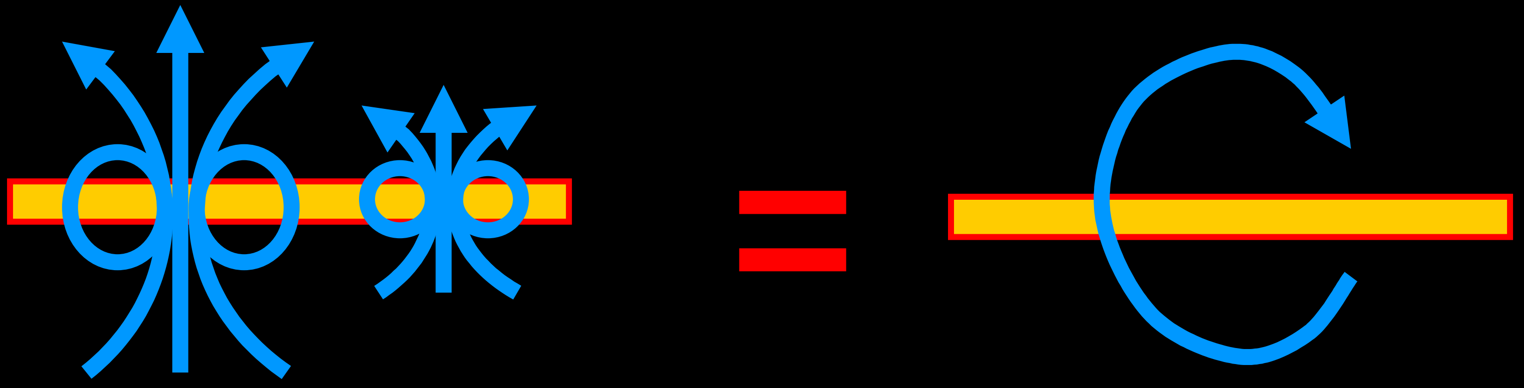
Single Layer



Double Layer



Double Layer-Vortex Relationship



The doublet is used for the lifting body cases, the source can not produce A vortex like effect!!!

Direct Integral Formulation

Potential Formulation

- This is the integral used in this work

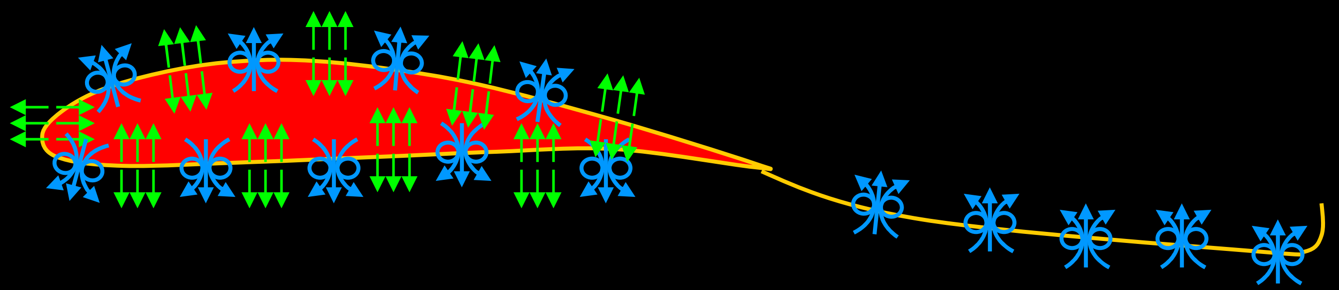
$$\phi(\vec{x}') = \frac{1}{4\pi}\int\int_{S_B}\left(\frac{\partial\phi}{\partial n}\frac{1}{||\vec{x}-\vec{x}'||}\right)dS_B - \frac{1}{4\pi}\int\int_{S_{B+W}}\left(\phi\frac{\partial}{\partial n}\left(\frac{1}{||\vec{x}-\vec{x}'||}\right)\right)dS_{B+W}$$

- Inserting the boundary conditions:

$$\phi(\vec{x}) = \frac{1}{4\pi}\int\int_{S_B} -\vec{V}_\infty\cdot\hat{n}_{\vec{x}}\frac{1}{||\vec{x}-\vec{x}'||}dS_B - \frac{1}{4\pi}\int\int_{S_{B+W}}\phi(\vec{x})\frac{\partial}{\partial n}\left(\frac{1}{||\vec{x}-\vec{x}'||}\right)dS_{B+W}$$

Math-to-English Translation

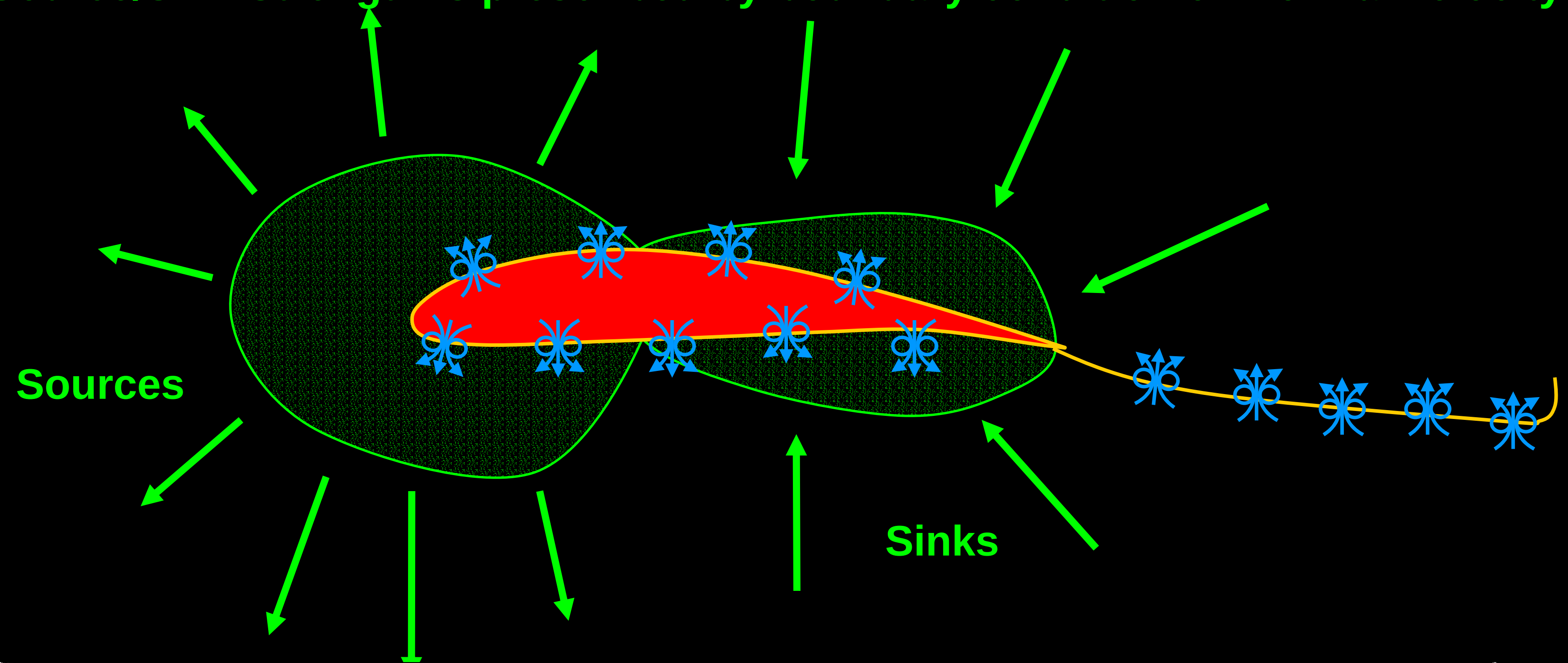
What Are We Trying To Do?



$$\phi(\vec{x}) = \frac{1}{4\pi}\int\int_{S_B} -\vec{V}_\infty \cdot \hat{n} \frac{1}{\|\vec{x}-\vec{x}'\|} dS_B - \frac{1}{4\pi}\int\int_{S_{B+W}} \phi(\vec{x}) \frac{\partial}{\partial n}\left(\frac{1}{\|\vec{x}-\vec{x}'\|}\right) dS_{B+W}$$

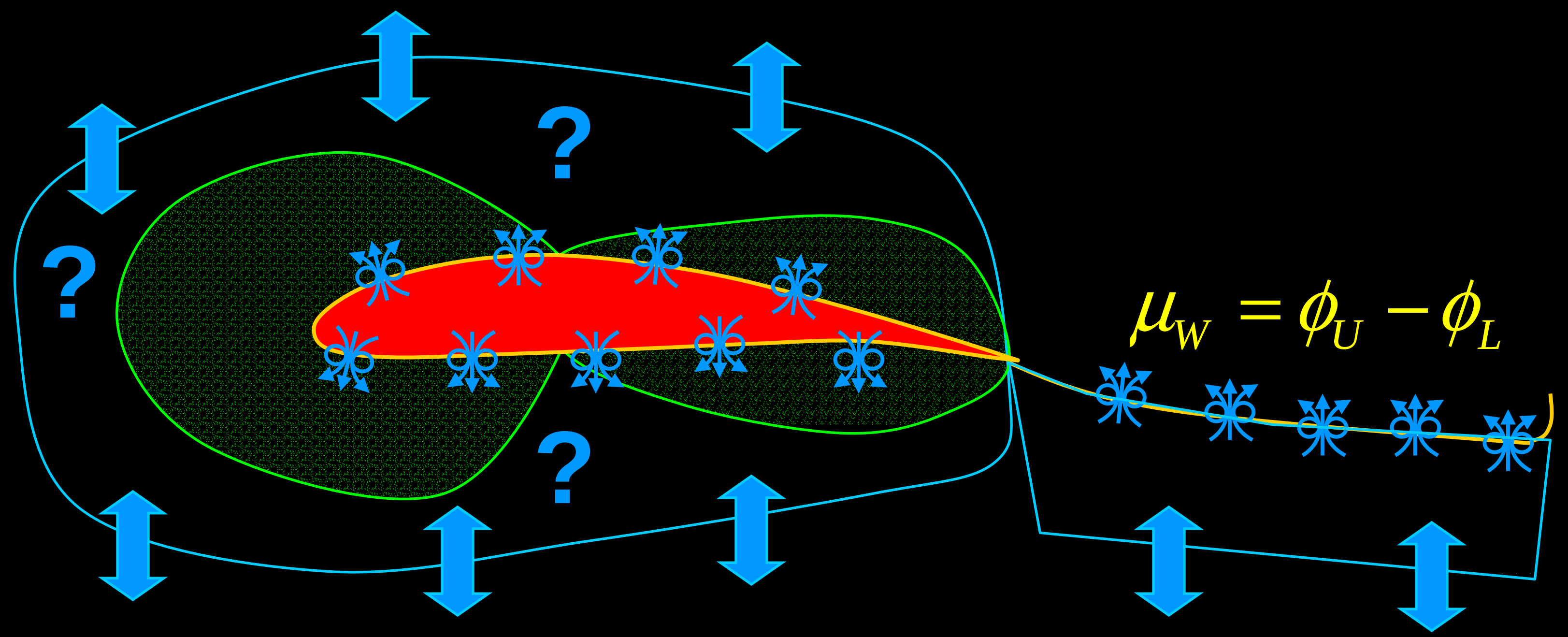
Prescribe the Single Layer

Source/Sink Strength is prescribed by boundary condition on normal velocity



$$\phi(\vec{x}) = \frac{1}{4\pi}\int\int_{S_B} -\vec{V}_\infty \cdot \hat{n}_{\vec{x}} \frac{1}{\|\vec{x}-\vec{x}'\|} dS_B - \frac{1}{4\pi}\int\int_{S_{B+W}} \phi(\vec{x}) \frac{\partial}{\partial n}\left(\frac{1}{\|\vec{x}-\vec{x}'\|}\right) dS_{B+W}$$

Solve For the Potential Distribution



$$\phi(\vec{x}) = \frac{1}{4\pi}\int\int_{S_B} -\vec{V}_\infty \cdot \hat{n}_{\vec{x}} \frac{1}{\|\vec{x}-\vec{x}'\|} dS_B - \frac{1}{4\pi}\int\int_{S_{B+W}} \phi(\vec{x}) \frac{\partial}{\partial n}\left(\frac{1}{\|\vec{x}-\vec{x}'\|}\right) dS_{B+W}$$

Direct Integral Formulation

Potential Formulation

- The direct potential formulation

$$\phi(\vec{x}) = \frac{1}{4\pi}\int\int_{S_B} -\vec{V}_\infty \cdot \hat{n}_{\vec{x}} \frac{1}{\|\vec{x}-\vec{x}'\|} dS_B - \frac{1}{4\pi}\int\int_{S_{B+W}} \phi(\vec{x}) \frac{\partial}{\partial n}\left(\frac{1}{\|\vec{x}-\vec{x}'\|}\right) dS_{B+W}$$

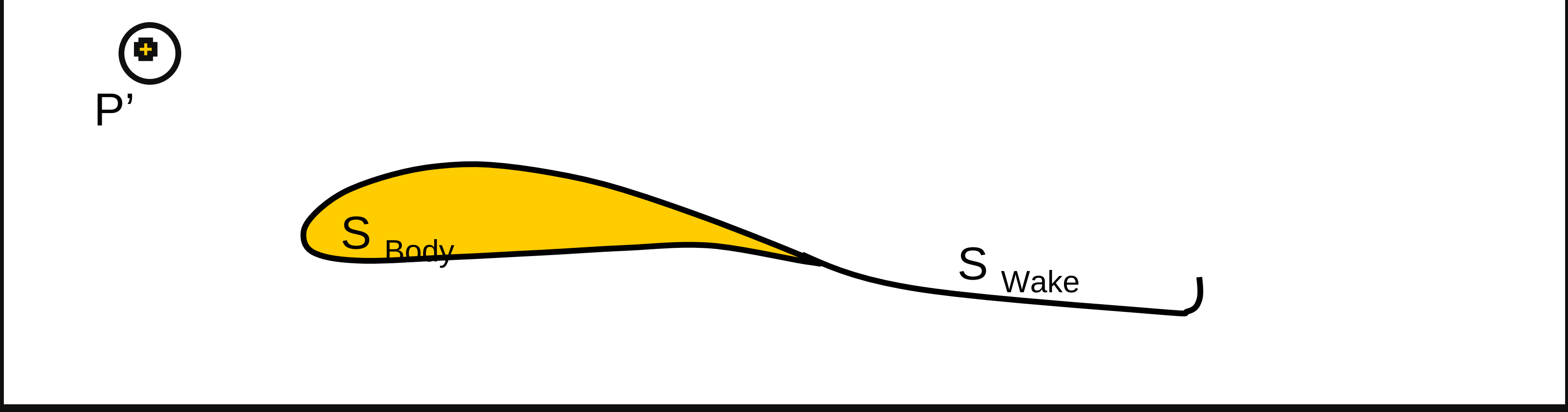
- Solve for the potential on the surface
  - Rapid post processing to get the surface velocities
  - The typical Laplace "type" problem formulation
  - NO distinction between lifting and non-lifting surfaces

The Indirect Approach

- Consider an inner and outer potential domain:

$$\phi(\vec{x}') = \frac{1}{4\pi}\int\int_{S_B}\left[\frac{\partial\phi}{\partial n}\right]_e \frac{1}{\|\vec{x}-\vec{x}'\|}dS_B - \frac{1}{4\pi}\int\int_{S_{B+W}}[\phi(\vec{x})]_e \frac{\partial}{\partial n}\left(\frac{1}{\|\vec{x}-\vec{x}'\|}\right)dS_{B+W}$$

$$0 = \frac{1}{4\pi}\int\int_{S_B}\left[\frac{\partial\phi}{\partial n}\right]_i \frac{1}{\|\vec{x}-\vec{x}'\|}dS_B - \frac{1}{4\pi}\int\int_{S_B}[\phi(\vec{x})]_i \frac{\partial}{\partial n}\left(\frac{1}{\|\vec{x}-\vec{x}'\|}\right)dS_B$$



The inner-outer Potential

- Subtracting the inner and outer potential equations:

$$\phi(\vec{x}') = \frac{1}{4\pi}\int\int_{S_B}\left(\left[\frac{\partial\phi}{\partial n}\right]_e - \left[\frac{\partial\phi}{\partial n}\right]_i\right)\frac{1}{\|\vec{x}-\vec{x}'\|}dS_B -$$

$$\frac{1}{4\pi}\int\int_{S_B}\left([\phi(\vec{x})]_e - [\phi(\vec{x})]_i\right)\frac{\partial}{\partial n}\left(\frac{1}{\|\vec{x}-\vec{x}'\|}\right)dS_B - \frac{1}{4\pi}\int\int_{S_W}[\phi(\vec{x})]_e\frac{\partial}{\partial n}\left(\frac{1}{\|\vec{x}-\vec{x}'\|}\right)dS_W$$

- We can define:

$$\sigma = \left[\frac{\partial\phi}{\partial n}\right]_e - \left[\frac{\partial\phi}{\partial n}\right]_i$$

Source Strength

$$\mu = [\phi(\vec{x})]_e - [\phi(\vec{x})]_i$$

Doublet Strength

Indirect Source Formulation

Fredholm Integral of the Second Kind

- The source singularity integral equation:

$$\phi(\vec{x}') = \frac{1}{4\pi} \int\int_{S_B} \sigma \frac{1}{||\vec{x} - \vec{x}'||} dS_B$$

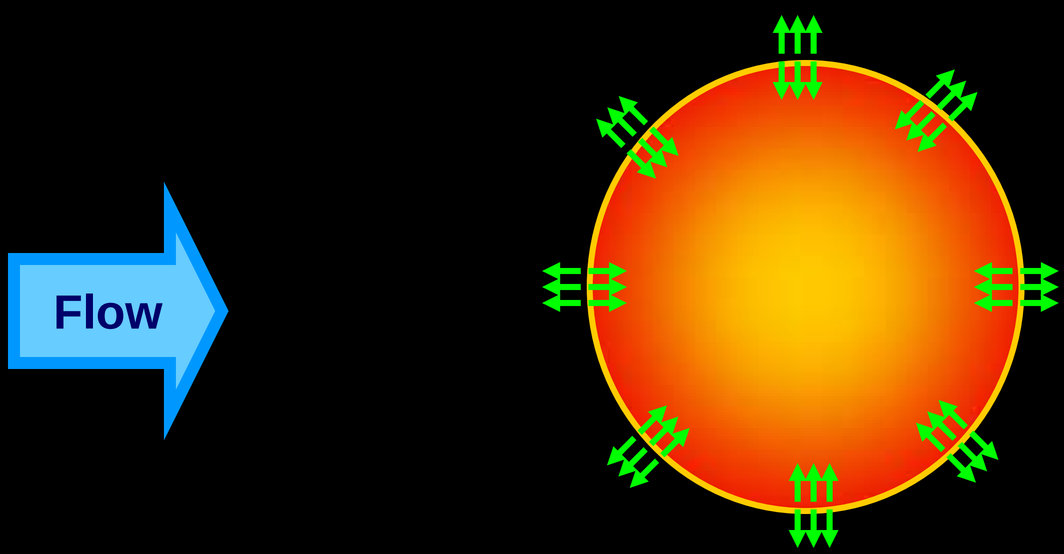
- We can exploit the Neumann B.C.'s by taking the gradient
  - Method explored in the unaccelerated method

$$-\vec{V}_\infty(\vec{x}') \cdot \hat{n}_{\vec{x}'} = \vec{v}(\vec{x}') \cdot \hat{n}_{\vec{x}'} = \nabla_{\vec{x}'}(\phi) \cdot \hat{n}_{\vec{x}'} = \frac{c\sigma(\vec{x}')}{2} + \frac{1}{4\pi} \nabla_{\vec{x}'} \cdot \hat{n}_{\vec{x}'} \oint_{S_B} \sigma \frac{1}{||\vec{x} - \vec{x}'||} dS_B$$

Physically

Source Velocity Formulation

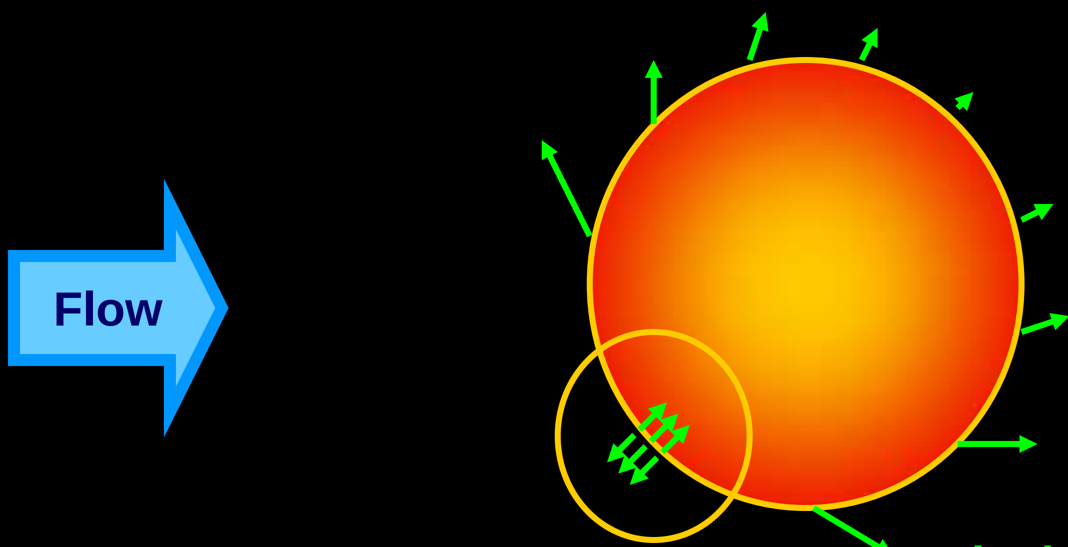
Smear Surface With Source Singularities



$$-\vec{V}_\infty(\vec{x}')\cdot\hat{n}_{\vec{x}'} = \vec{v}(\vec{x}')\cdot\hat{n}_{\vec{x}'} = \nabla_{\vec{x}'}(\phi)\cdot\hat{n}_{\vec{x}'} = \frac{c\sigma(\vec{x}')}{2} + \frac{1}{4\pi}\nabla_{\vec{x}'}\cdot\hat{n}_{\vec{x}'}\oint_{S_B}\sigma\frac{1}{\|\vec{x}-\vec{x}'\|}dS_B$$

$$-\vec{V}_\infty(\vec{x}')\cdot\hat{n}_{\vec{x}'} = \vec{v}(\vec{x}')\cdot\hat{n}_{\vec{x}'} = \nabla_{\vec{x}'}(\phi)\cdot\hat{n}_{\vec{x}'} = \frac{c\sigma(\vec{x}')}{2} + \frac{1}{4\pi}\nabla_{\vec{x}'}\cdot\hat{n}_{\vec{x}'}\oint_{S_B}\sigma\frac{1}{\|\vec{x}-\vec{x}'\|}dS_B$$

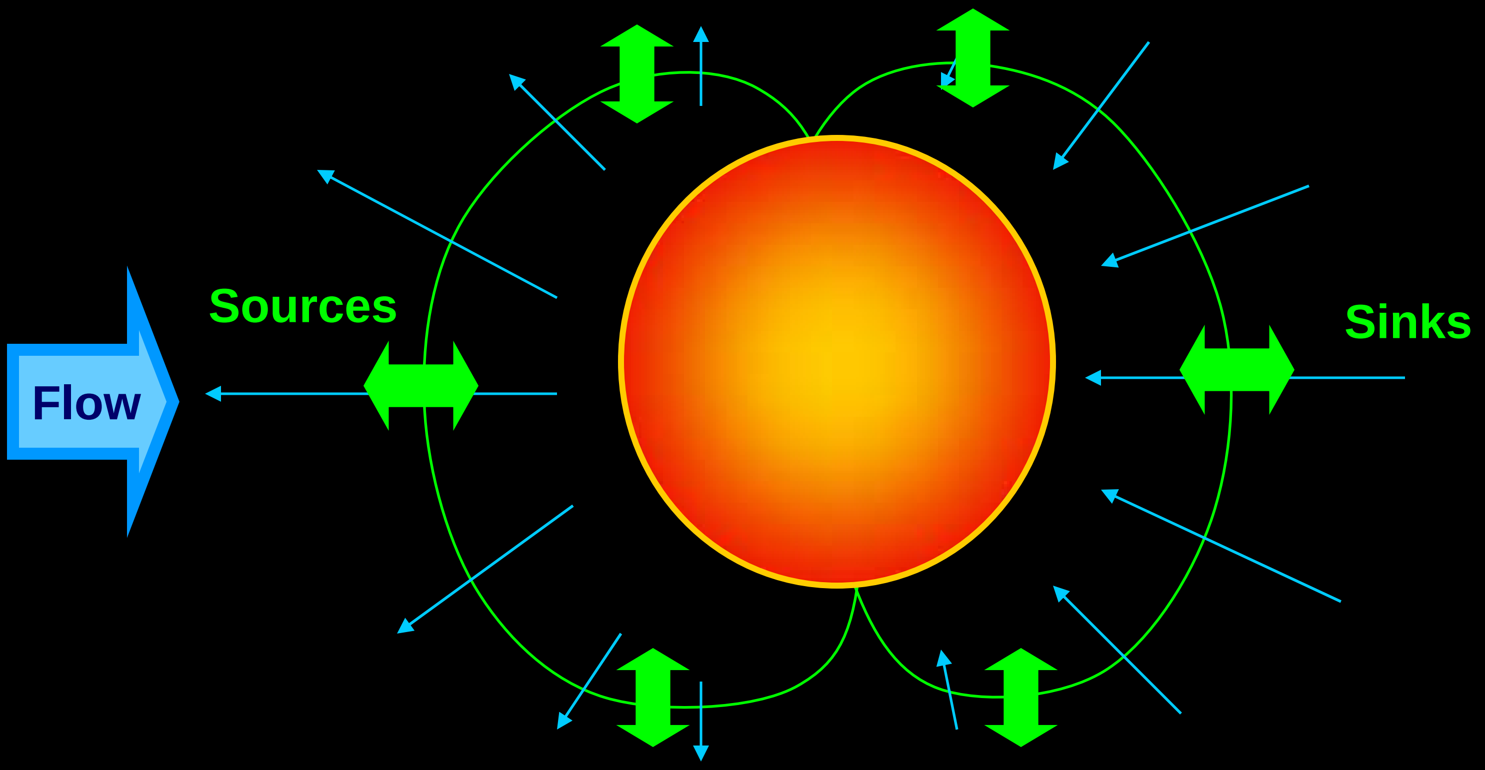
Each Singularity produces a velocity field corresponding to the singularity



Velocity Field Due to the Source

$$-\vec{V}_\infty(\vec{x}') \cdot \hat{n}_{\vec{x}'} = \vec{v}(\vec{x}') \cdot \hat{n}_{\vec{x}'} = \nabla_{\vec{x}'}(\phi) \cdot \hat{n}_{\vec{x}'} = \frac{c\sigma(\vec{x}')}{2} + \frac{1}{4\pi} \nabla_{\vec{x}'} \cdot \hat{n}_{\vec{x}'} \oint_{S_B} \sigma \frac{1}{\|\vec{x} - \vec{x}'\|} dS_B$$

Adjust the strengths of the sources over the surface to achieve a zero velocity through the surface



Indirect Dipole Formulation

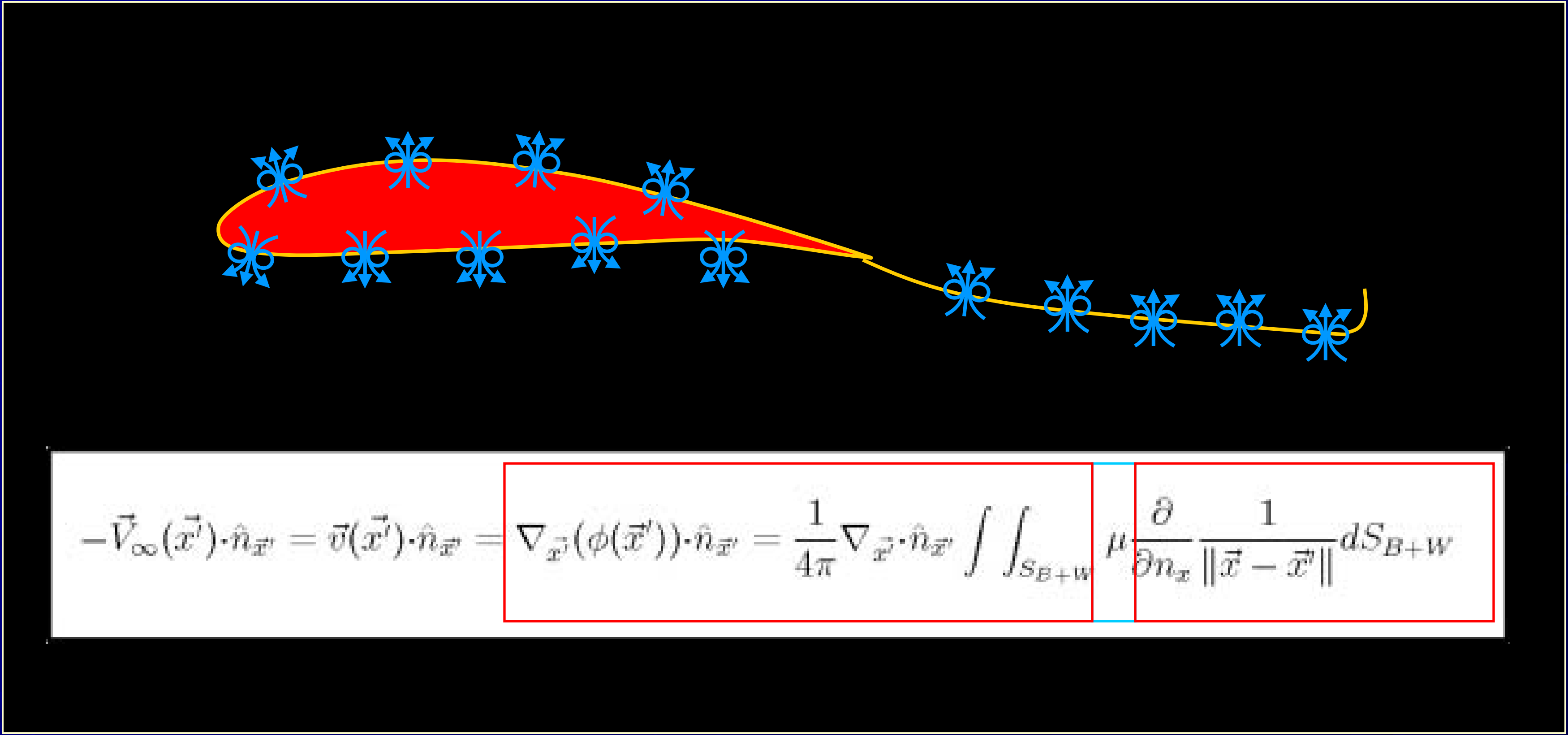
Hypersingular Self Term Evaluation

- The second formulation sets the source strength to zero.
  - Unaccelerated method uses this

$$-\vec{V}_\infty(\vec{x}')\cdot\hat{n}_{\vec{x}'} = \vec{v}(\vec{x}')\cdot\hat{n}_{\vec{x}'} = \nabla_{\vec{x}'}(\phi(\vec{x}'))\cdot\hat{n}_{\vec{x}'} = \frac{1}{4\pi}\nabla_{\vec{x}'}\cdot\hat{n}_{\vec{x}'}\int\int_{S_{B+W}}\mu\frac{\partial}{\partial n_x}\frac{1}{\|\vec{x}-\vec{x}'\|}dS_{B+W}$$

Physically

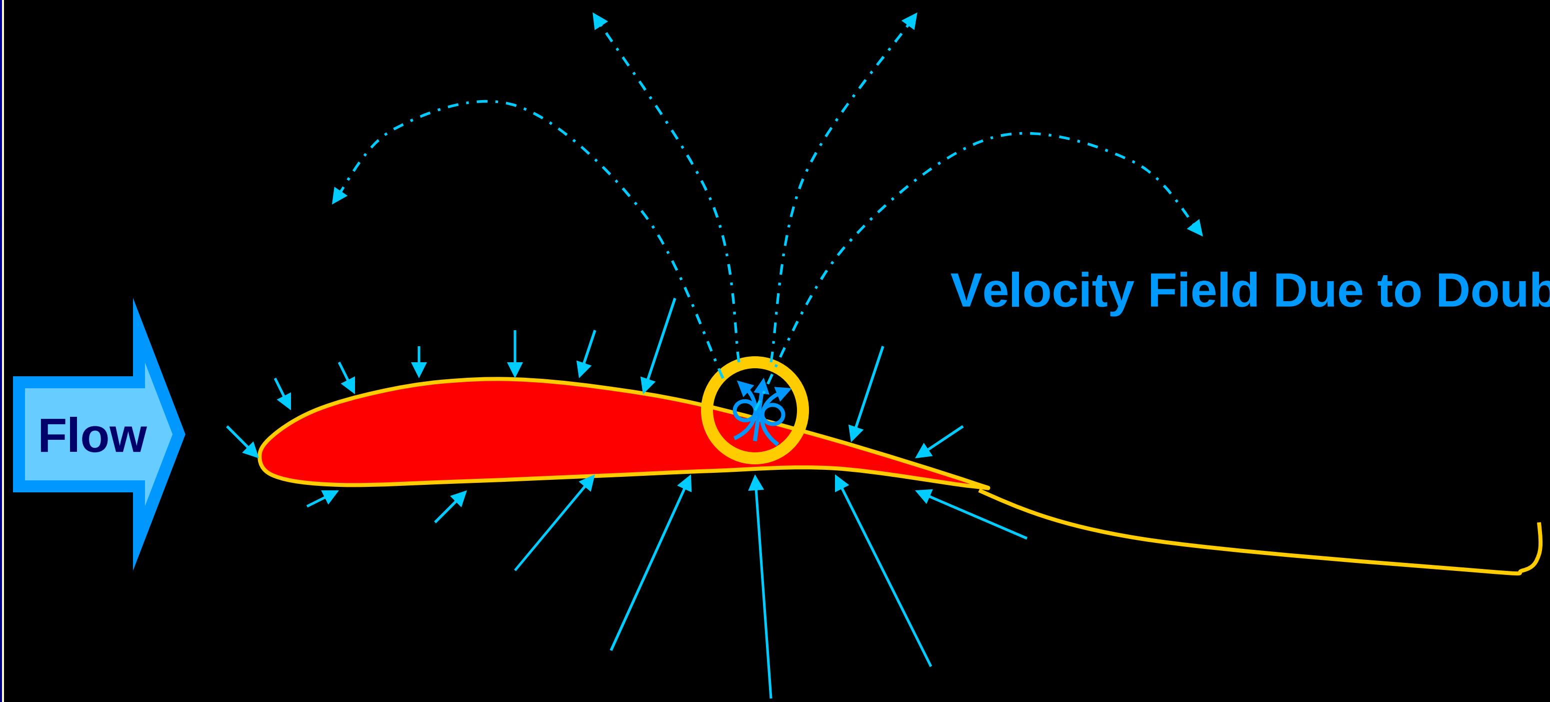
What Are We Trying To Do?



Physically

Doublet Velocity Formulation

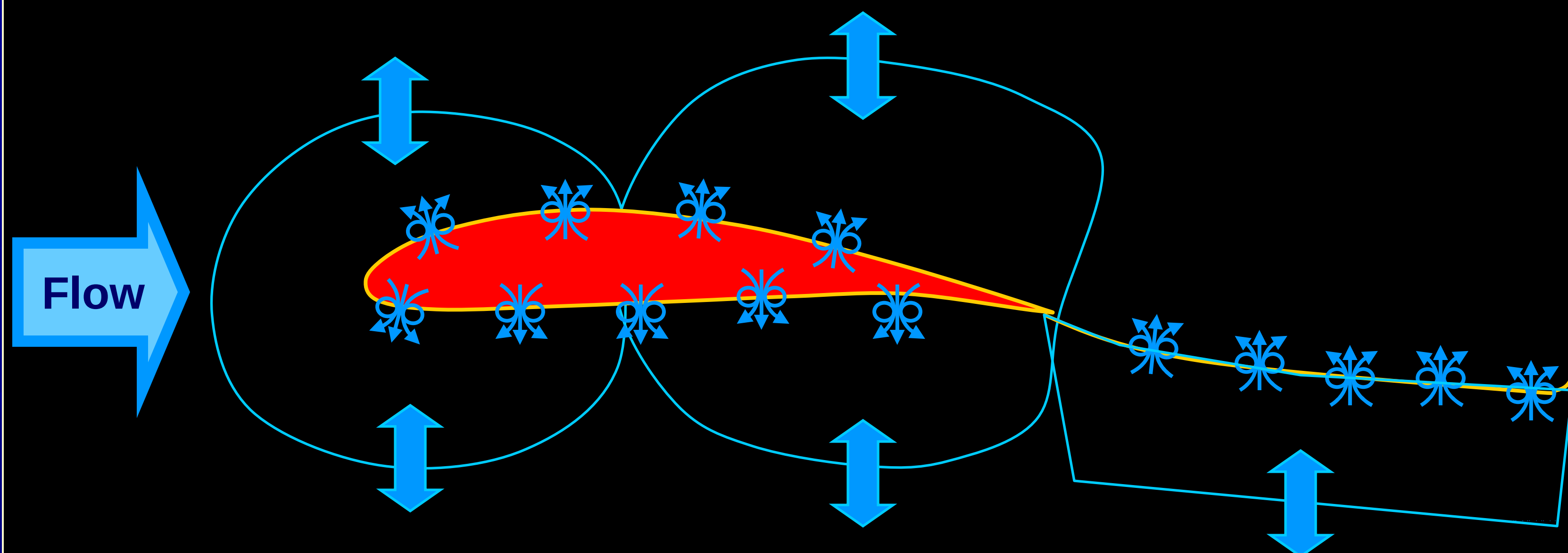
Each Singularity produces a velocity field corresponding to the singularity



$$-\vec{V}_{\infty}(\vec{x}')\cdot\hat{n}_{\vec{x}'} = \vec{v}(\vec{x}')\cdot\hat{n}_{\vec{x}'} = \nabla_{\vec{x}'}(\phi(\vec{x}'))\cdot\hat{n}_{\vec{x}'} = \frac{1}{4\pi}\nabla_{\vec{x}'}\cdot\hat{n}_{\vec{x}'}\int\int_{S_{B+W}} \mu\frac{\partial}{\partial n_{x}}\frac{1}{\|\vec{x}-\vec{x}'\|}dS_{B+W}$$

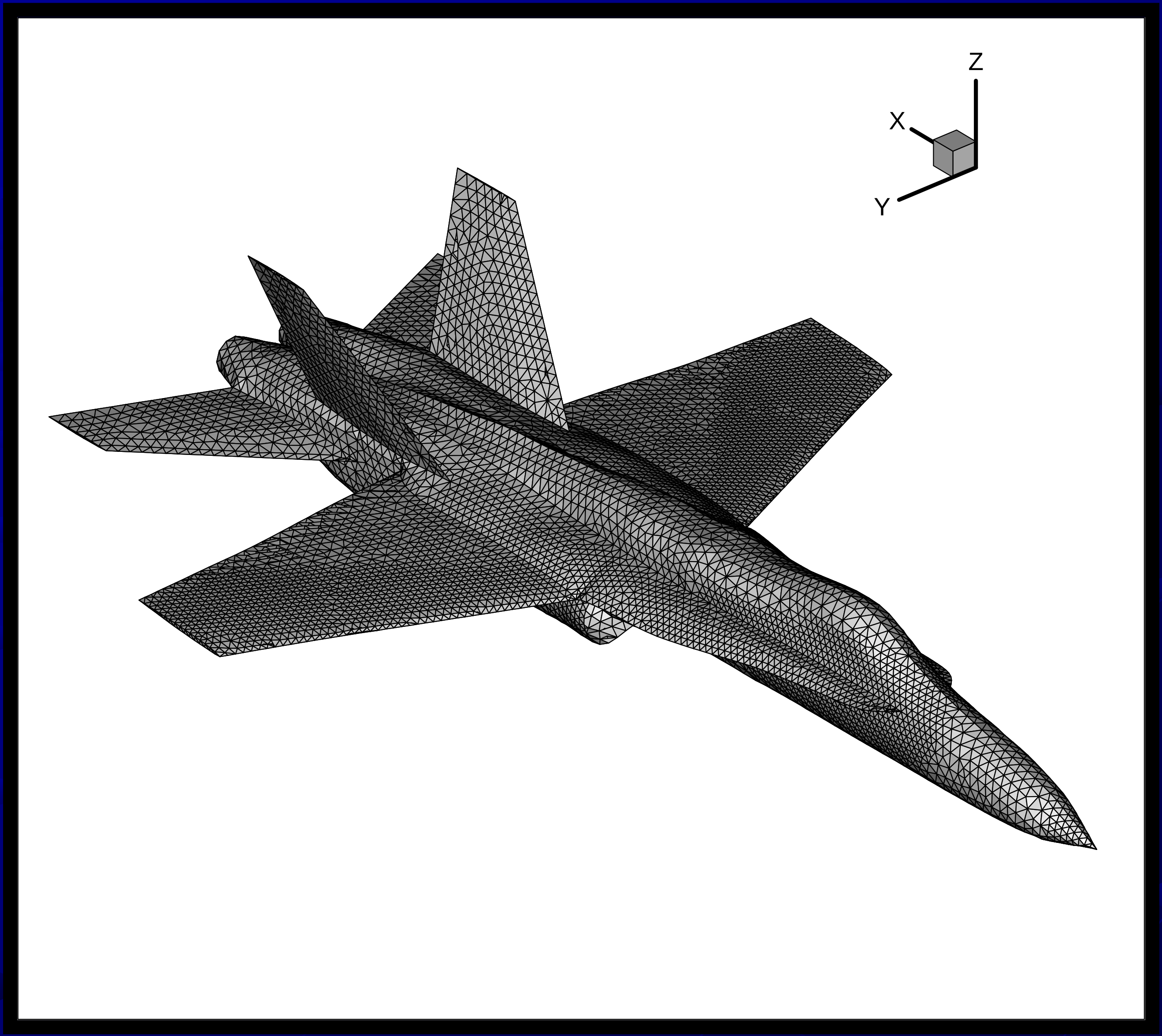
$$-\vec{V}_\infty(\vec{x}')\cdot\hat{n}_{\vec{x}'} = \vec{v}(\vec{x}')\cdot\hat{n}_{\vec{x}'} = \nabla_{\vec{x}'}(\phi(\vec{x}'))\cdot\hat{n}_{\vec{x}'} = \frac{1}{4\pi}\nabla_{\vec{x}'}\cdot\hat{n}_{\vec{x}'}\int\int_{S_{B+W}} \mu\frac{\partial}{\partial n_x}\frac{1}{\|\vec{x}-\vec{x}'\|}dS_{B+W}$$

Adjust Singularity Strength to get zero through-velocity at surface



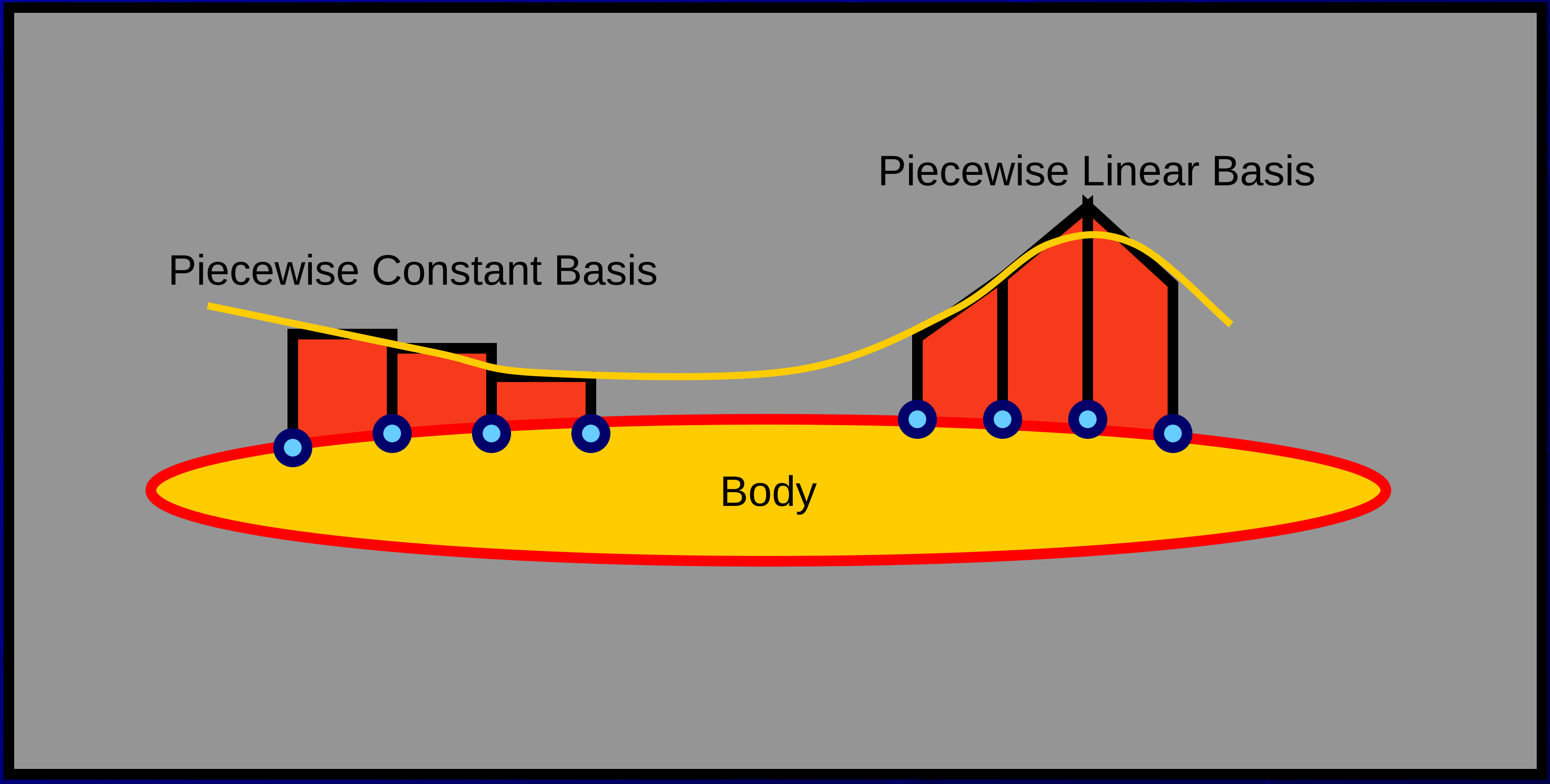
Discrete Implementation BEM

Discrete Geometry

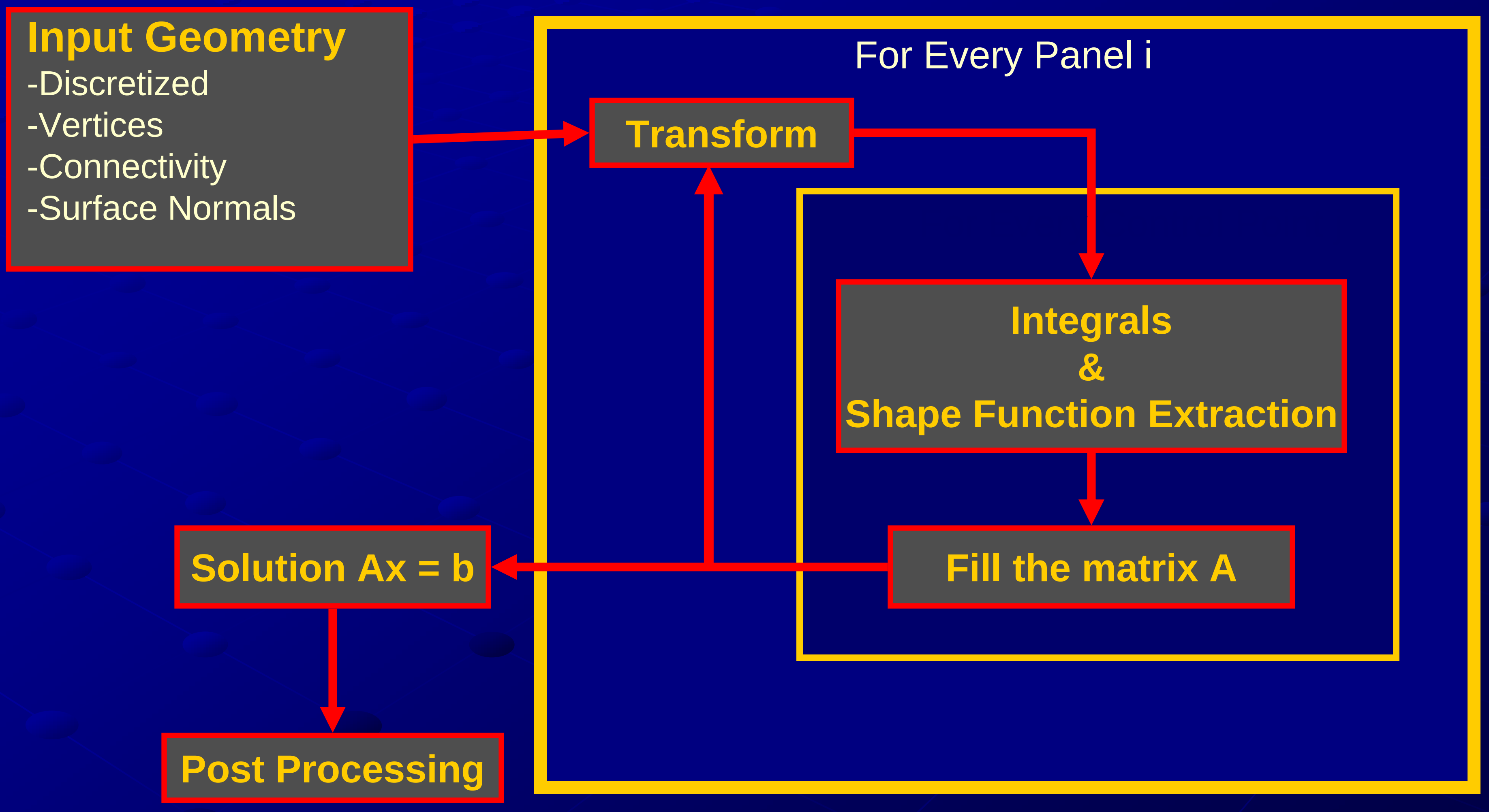


Computer Solution

Discretizing the equations (in 2-D):

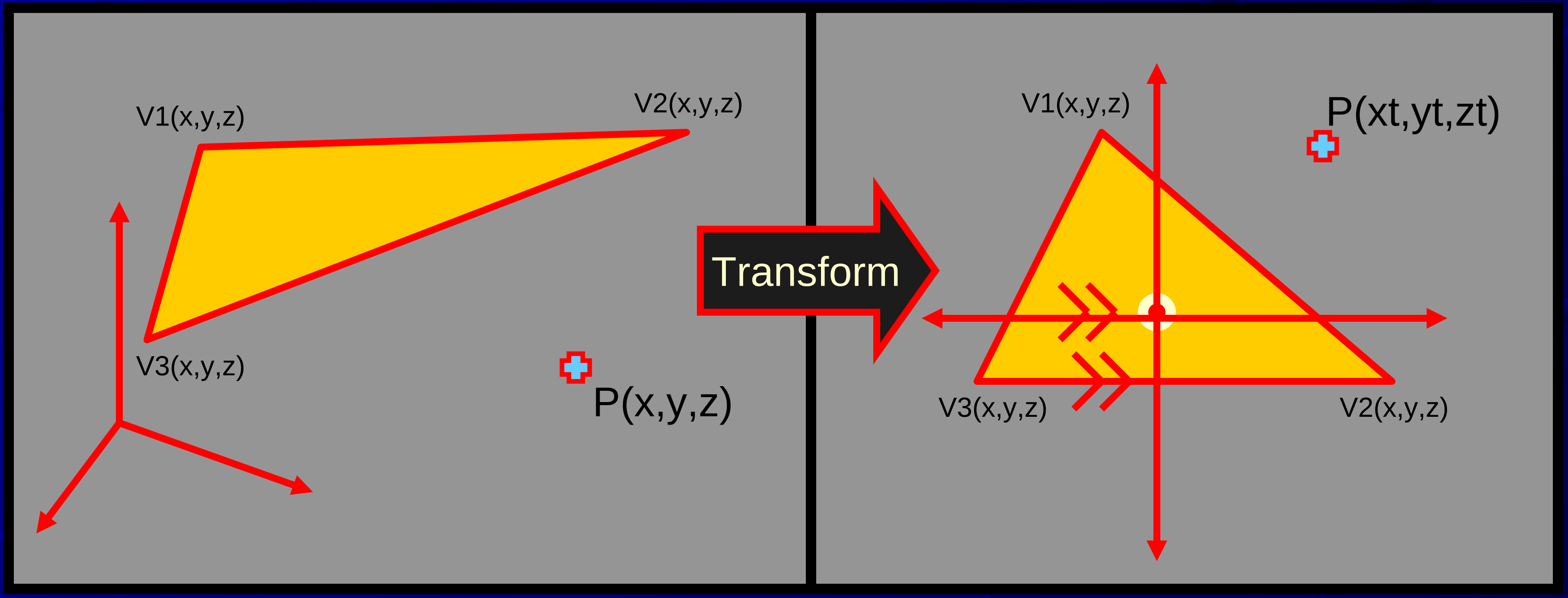


Direct Algorithm Flow Chart



Transformation

- From Arbitrary 3D to Panel based (~2D):



Panel Integrals

- Two Approaches:
  - Hess and Smith
    - Douglas Aircraft Co. ~c. 1960’s
  - Newman
    - MIT OE, ~c. 1985
- Double layer is the building block
  - Once evaluated, all other integrals are similar.

Hess & Smith: Constant Source Calculation

Hess, J.L., & Smith, A.M.O., “Calculation Of Potential flows about arbitrary bodies”, 1967.

The integral :

$$\phi(\vec{x}') = \int\int_S \frac{1}{\|\vec{x} - \vec{x}'\|} dS$$

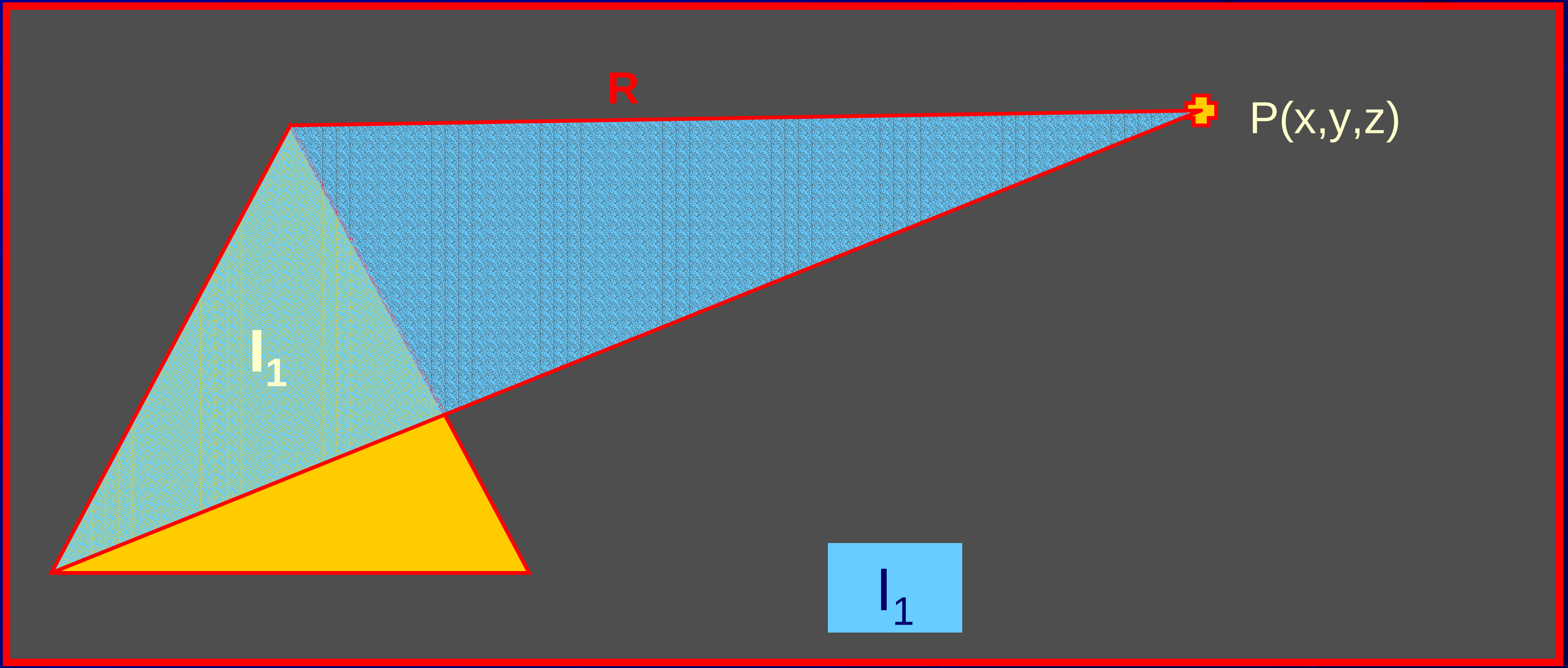
$$(\|x - x'\| = \sqrt{(R^2 + h^2)})$$

Is modified to an “in plane” integration:

$$\phi = \oint_{Perimeter} \int_0^R \frac{R dR d\theta}{\sqrt{(R^2 + z^2)}}$$

Hess and Smith

- Hess and Smith (Douglas A/C Company), suggest an in plane evaluation:



Hess and Smith

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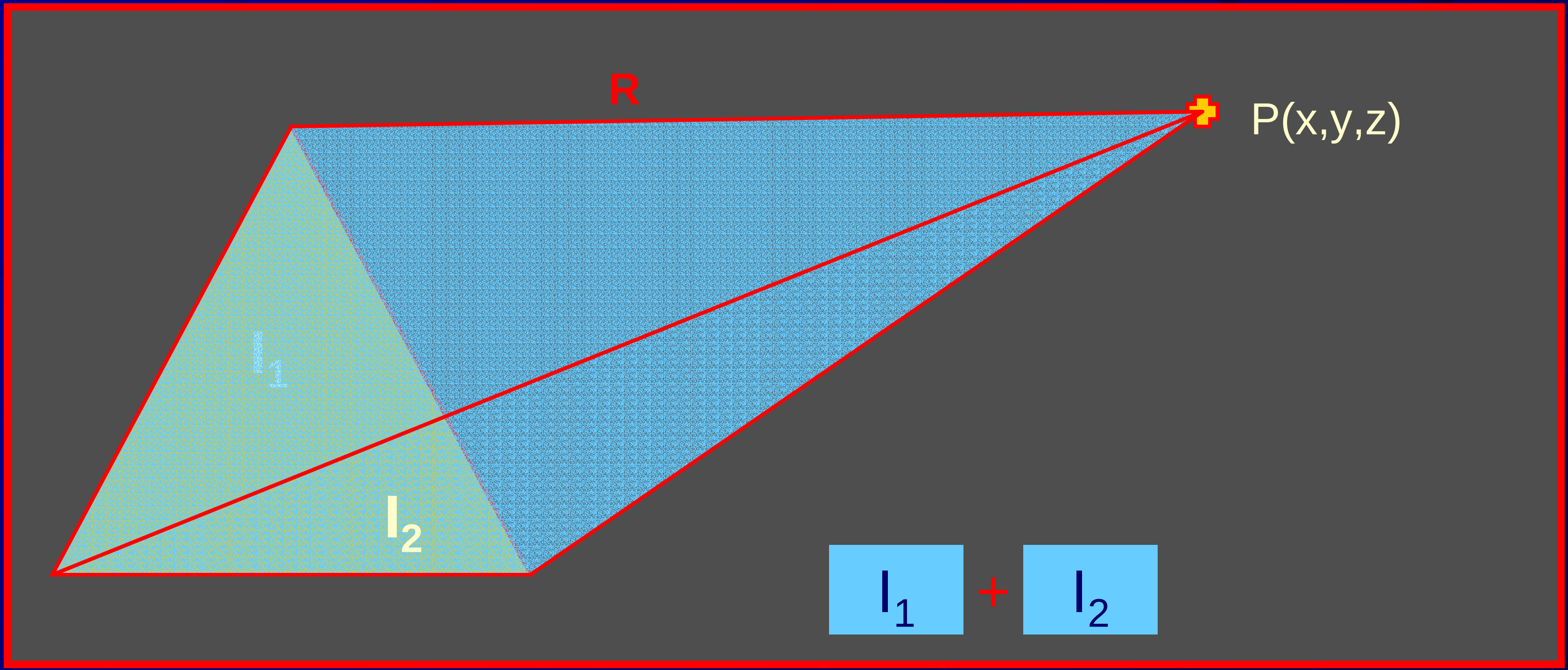

Hess and Smith

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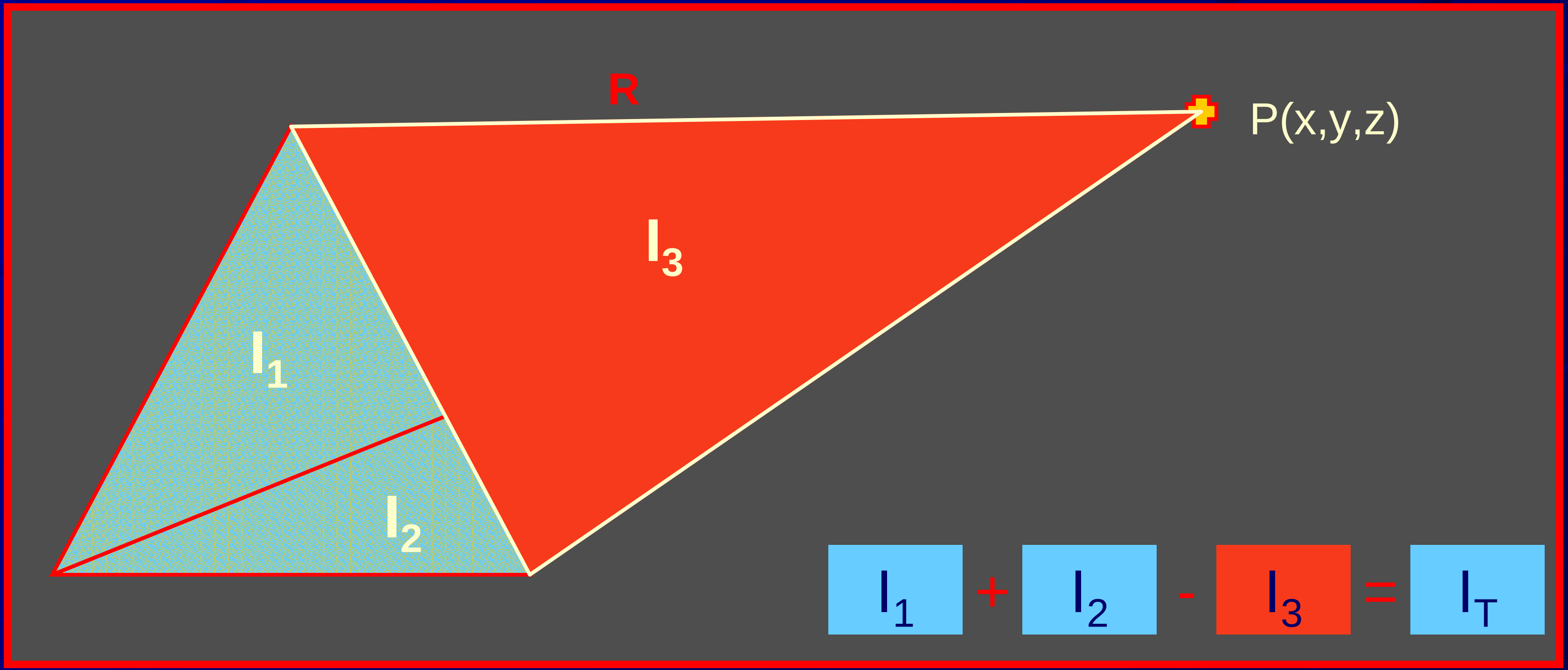


Hess and Smith Const. Doublet

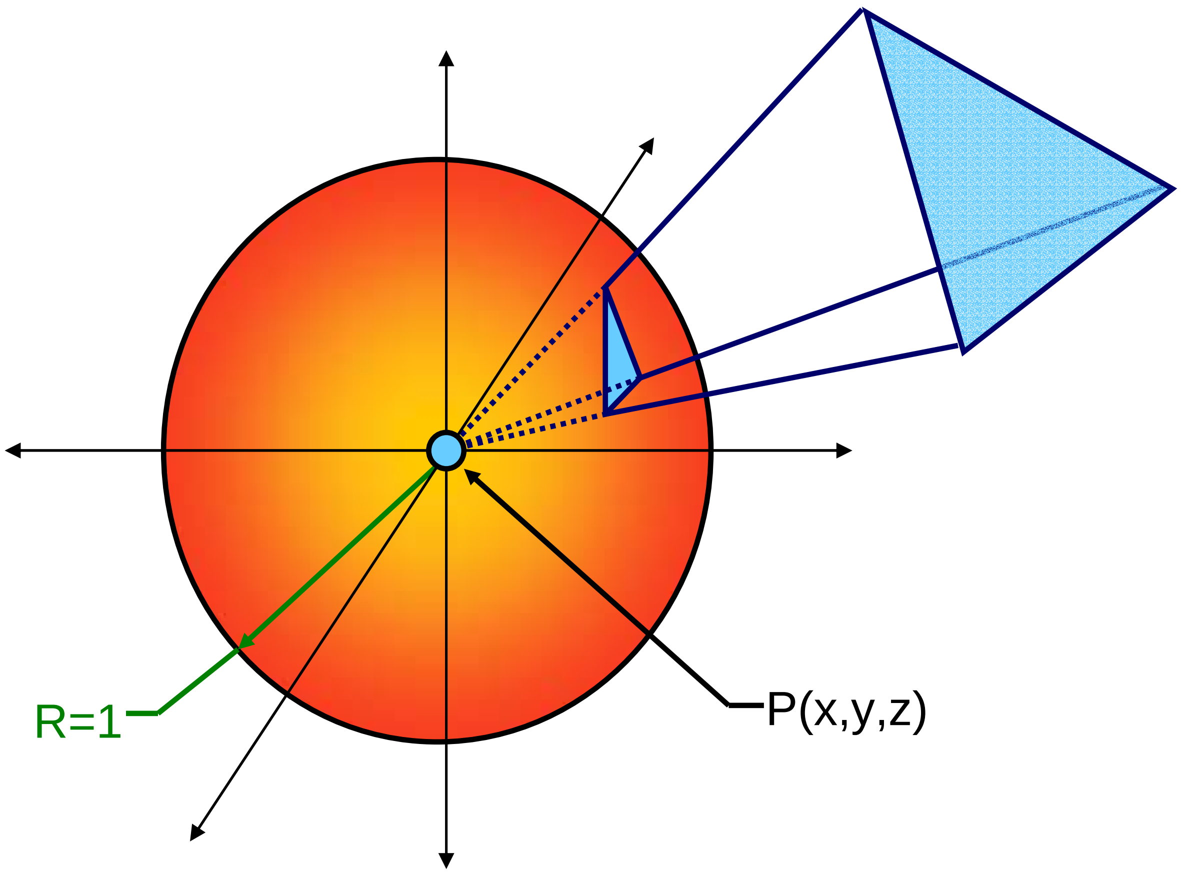
- Resulting expression for the constant strength doublet.
  - Complicated!!!

$$\psi^{c}_{H.S}(\xi^{T},\eta^{T},z'^{T}) = \sum_{i=1}^{i=NV} \left(arctan \left[\frac{\frac{l_i^{\eta}}{l_i^{\xi}} \cdot \left((\Delta\xi_i^{ST})^2 + (\Delta z_i'^{ST})^2 \right) - (\Delta\xi_i^{ST})(\Delta\eta_i^{ST})}{(\Delta z_i'^{ST}) \cdot \sqrt{(\Delta\xi_i^{ST})^2 + (\Delta\eta_i^{ST})^2 + (\Delta z_i'^{ST})}} \right] - \right.$$

$$\left. \sum_{i=1}^{i=NV} arctan \left[\frac{\frac{l_i^{\eta}}{l_i^{\xi}} \cdot \left((\Delta\xi_{i+1}^{ST})^2 + (\Delta z_{i+1}'^{ST})^2 \right) - (\Delta\xi_{i+1}^{ST})(\Delta\eta_{i+1}^{ST})}{(\Delta z_{i+1}'^{ST}) \cdot \sqrt{(\Delta\xi_{i+1}^{ST})^2 + (\Delta\eta_{i+1}^{ST})^2 + (\Delta z_{i+1}'^{ST})}} \right] \right)$$

Newman: Computing The Dipole Integral

Uses the Gauss-Bonnet Concept—projecting the panel onto a unit sphere, and determining the solid angle from sum of included angles:



.N.Newman, "Distributions Of Sources and Normal Dipoles over a Quadrilateral Panel", 1985

Newman Doublet

$$s_i^a = l_i^\eta \cdot \left((\Delta\xi_i^{ST})^2 + (\Delta z_0'^{ST})^2 - l_i^\xi \cdot \Delta\xi_i^{ST} \cdot \Delta\eta_i^{ST} \right)$$

$$s_i^b = l_i^\eta \cdot \left((\Delta\xi_{i+1}^{ST})^2 + (\Delta z_0'^{ST})^2 - l_i^\xi \cdot \Delta\xi_{i+1}^{ST} \cdot \Delta\eta_{i+1}^{ST} \right)$$

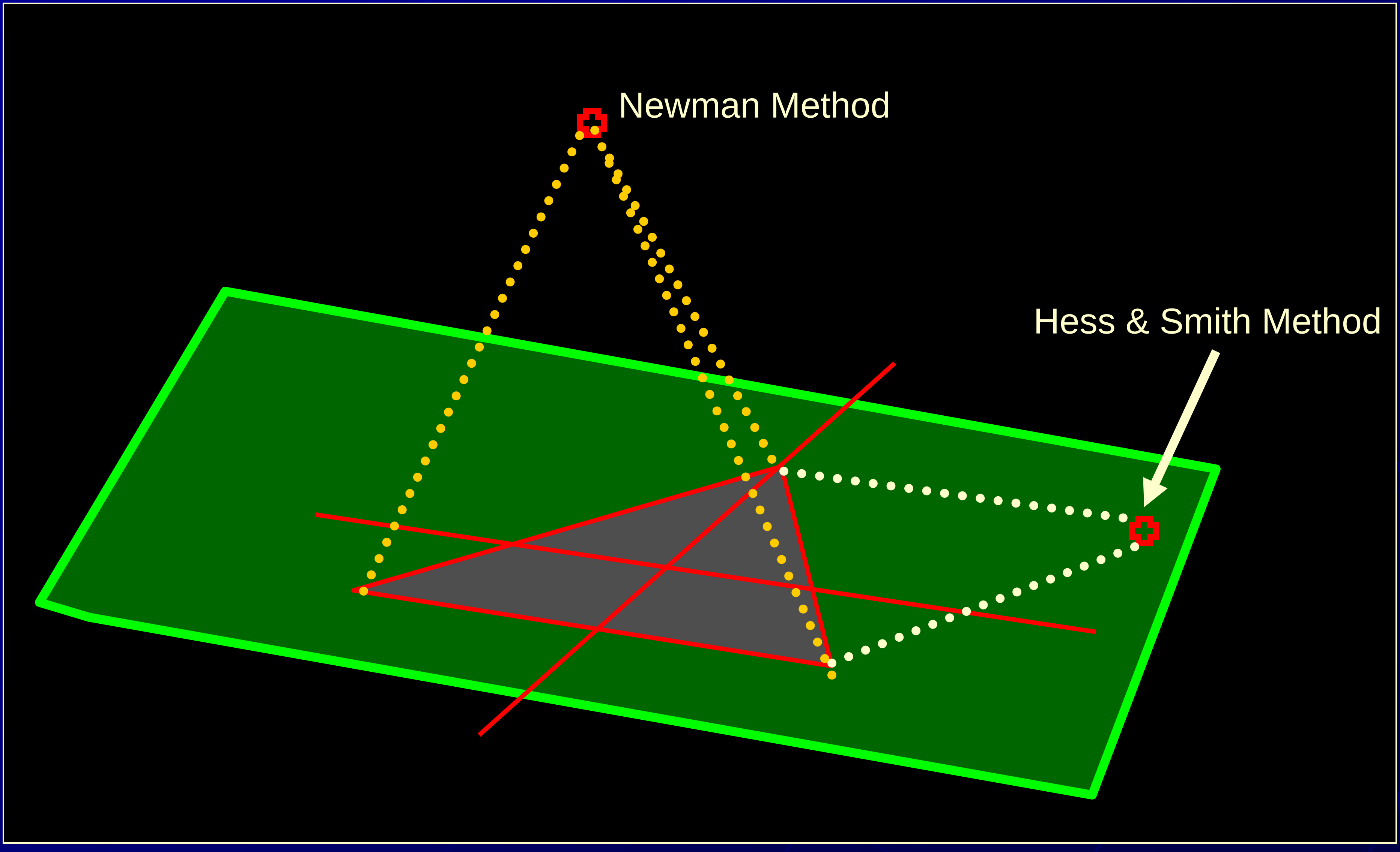
$$c_i^a = R_i^I \cdot \Delta z_0'^{ST} \cdot l_i^\xi$$

$$c_i^b = R_{i+1}^I \cdot \Delta z_0'^{ST} \cdot l_i^\xi$$

The final result is:

$$\psi_N^c(\xi^T, \eta^T, z'^T) = -\sum_{i=1}^{i=NV} arctan \left(\frac{s_i^a \cdot c_i^b - s_i^b \cdot c_i^a}{c_i^a \cdot c_i^b + s_i^a \cdot s_i^b} \right)$$

Hess & Smith Vs. Newman



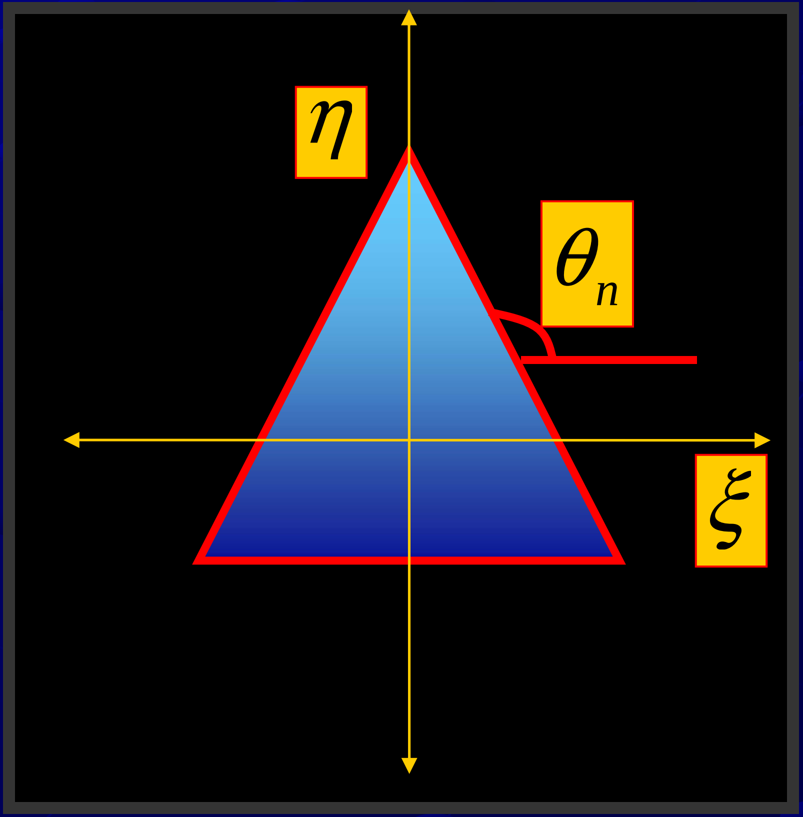
Calculating the Linear Variation Integrals

- General methods for the linear strength

$$\begin{pmatrix} \psi_{\bar{\xi}} \\ \psi_{\bar{\eta}} \end{pmatrix} = \iint_{S_{Panel}} \begin{pmatrix} \xi \\ \eta \end{pmatrix} \frac{d\xi d\eta}{r}$$

*

$$\begin{pmatrix} \psi_{\bar{\xi}} \\ \psi_{\bar{\eta}} \end{pmatrix} = \begin{pmatrix} \bar{\xi} \\ \bar{\eta} \end{pmatrix} \psi_{Const.}\, \mu \sum_{n=1}^{N.V} P_n \begin{pmatrix} \sin\theta_n \\ \cos\theta_n \end{pmatrix}$$



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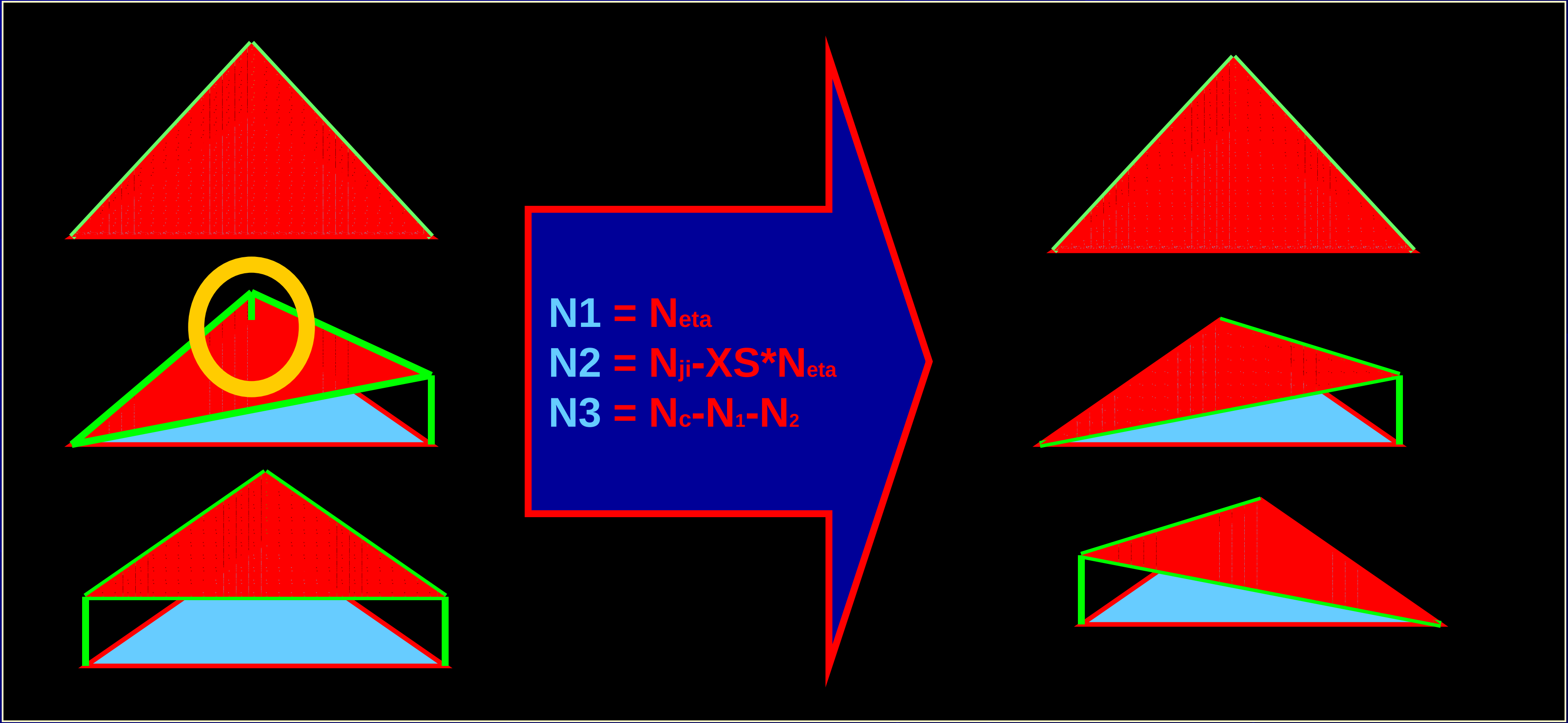
Linear Shape Functions

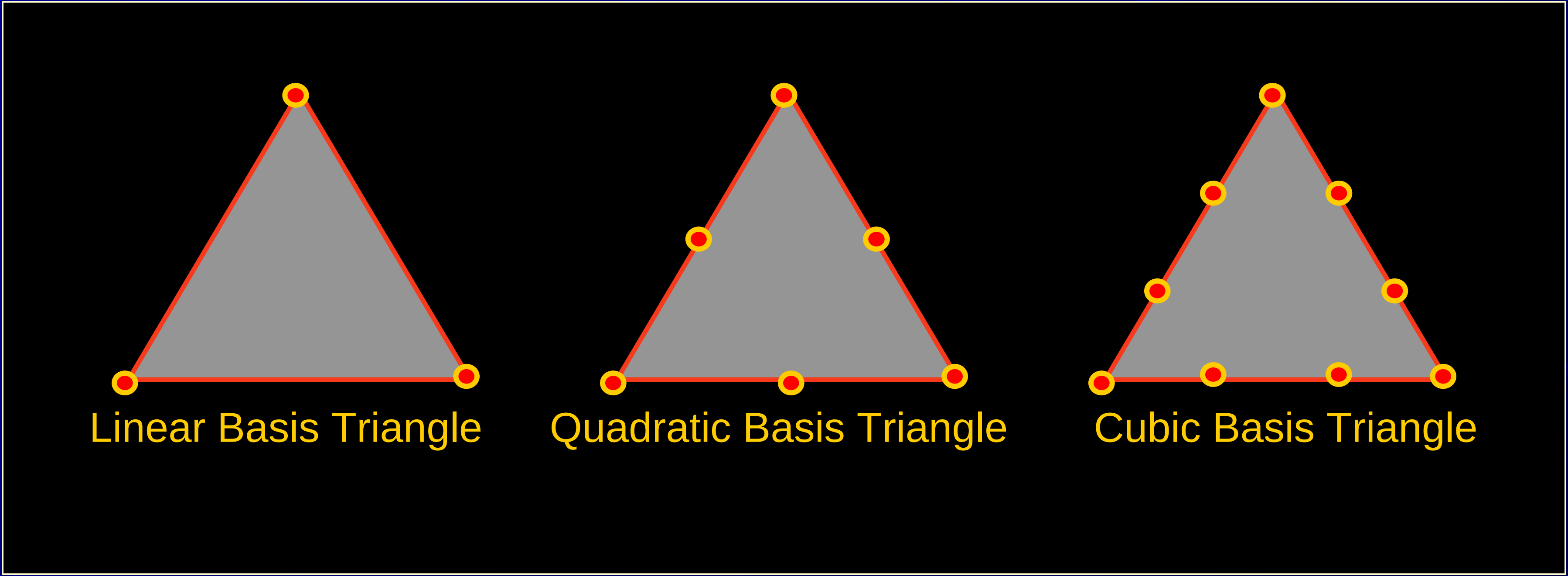
- Integrals
  - In N_{eta}, N_{ji}, N_{c} vs. N_1, N_2, N_3 system



Higher Order Shape Functions

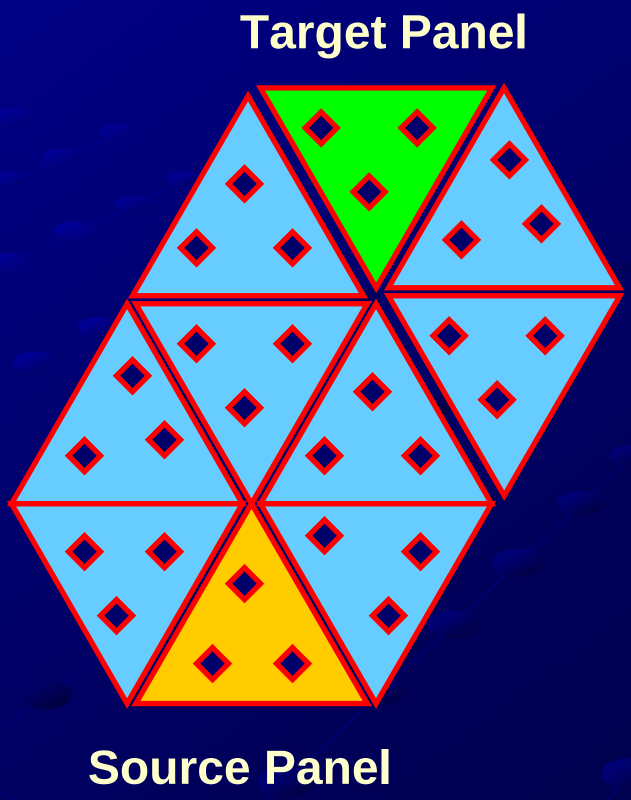
- The Linear Shape function is easily extended to higher order
  - Quadratic
  - Cubic … etc.
- This also may facilitate curved panel integration as suggested by Wang. et al.

Higher Order Triangles

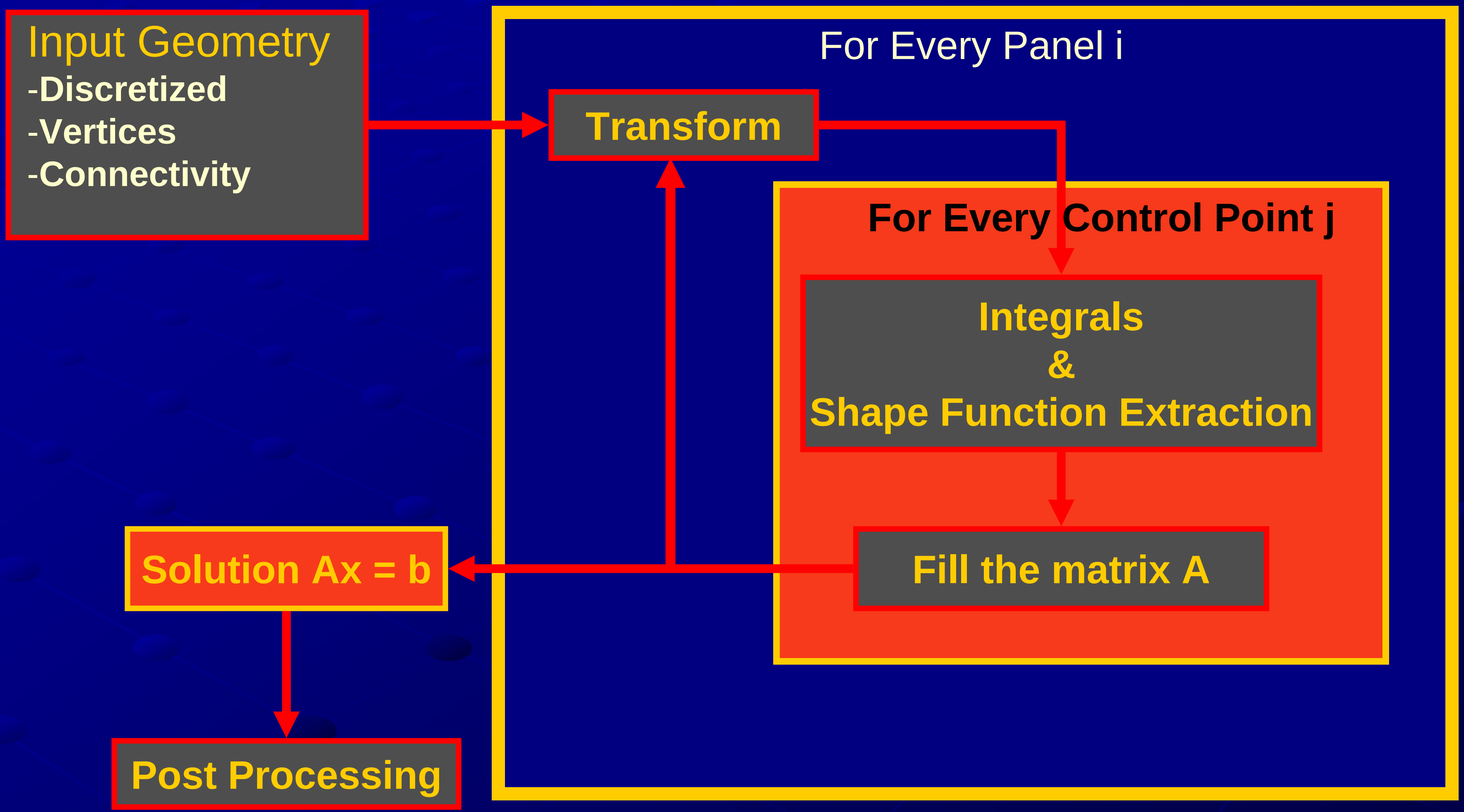


Galerkin

- Galerkin Formulation
  - Advantages:
    - Corners Are Easy
    - More Accurate
  - Disadvantages:
    - More Evaluation Pts.
      - Scales With NP
  - Numerical Evaluation of outer integral is necessary- G.Q.



Direct Algorithm Summary



Direct Solution

- Gaussian Elimination
  - Costly solution
    - Time $O(N^3)$
    - Memory $O(N^2)$
- N x N Interaction Integrals
  - **Direct interactions are time consuming:**
    - sin, cos, and log functions.

NEED SOMETHING MORE EFFICIENT

pFFT++

Matrix Vector Product Accelerator

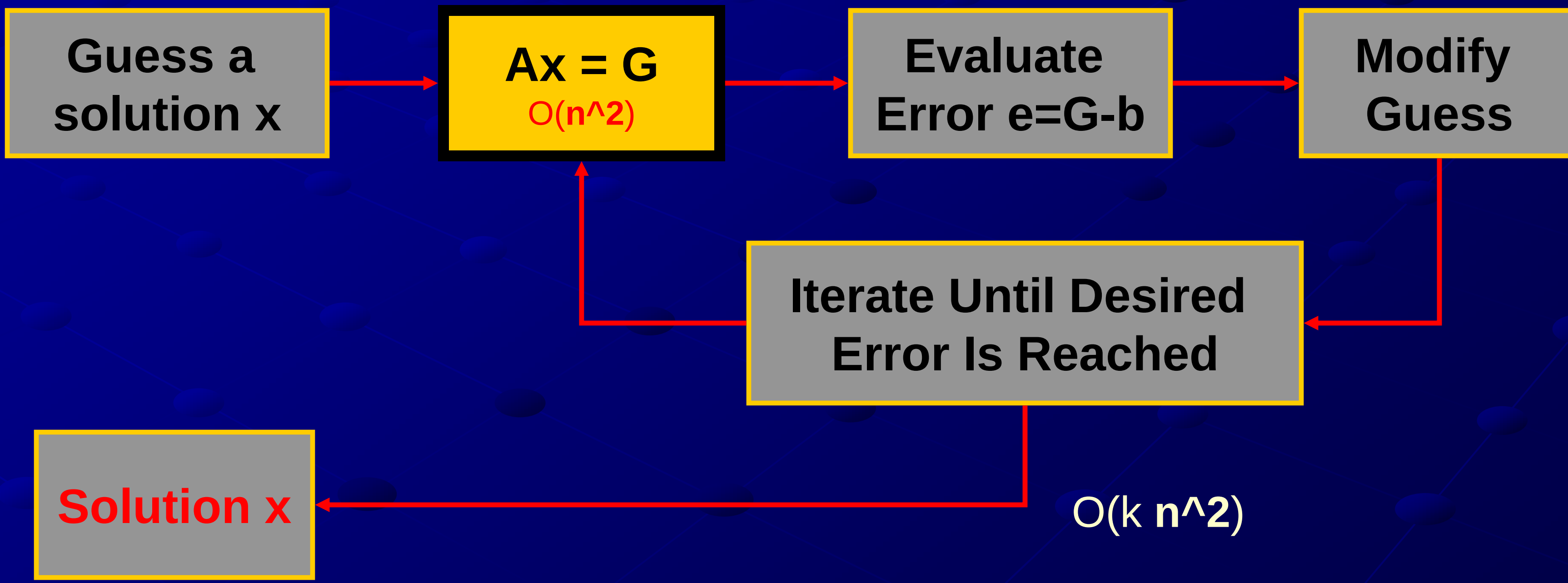
pFFT Acceleration

pFFT++ Implemented From

White, Zhu and Song Constant Collocation Code

Iterative Methods

- GMRES, GCR.
- Basic Idea of these methods:

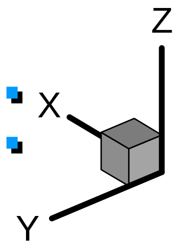


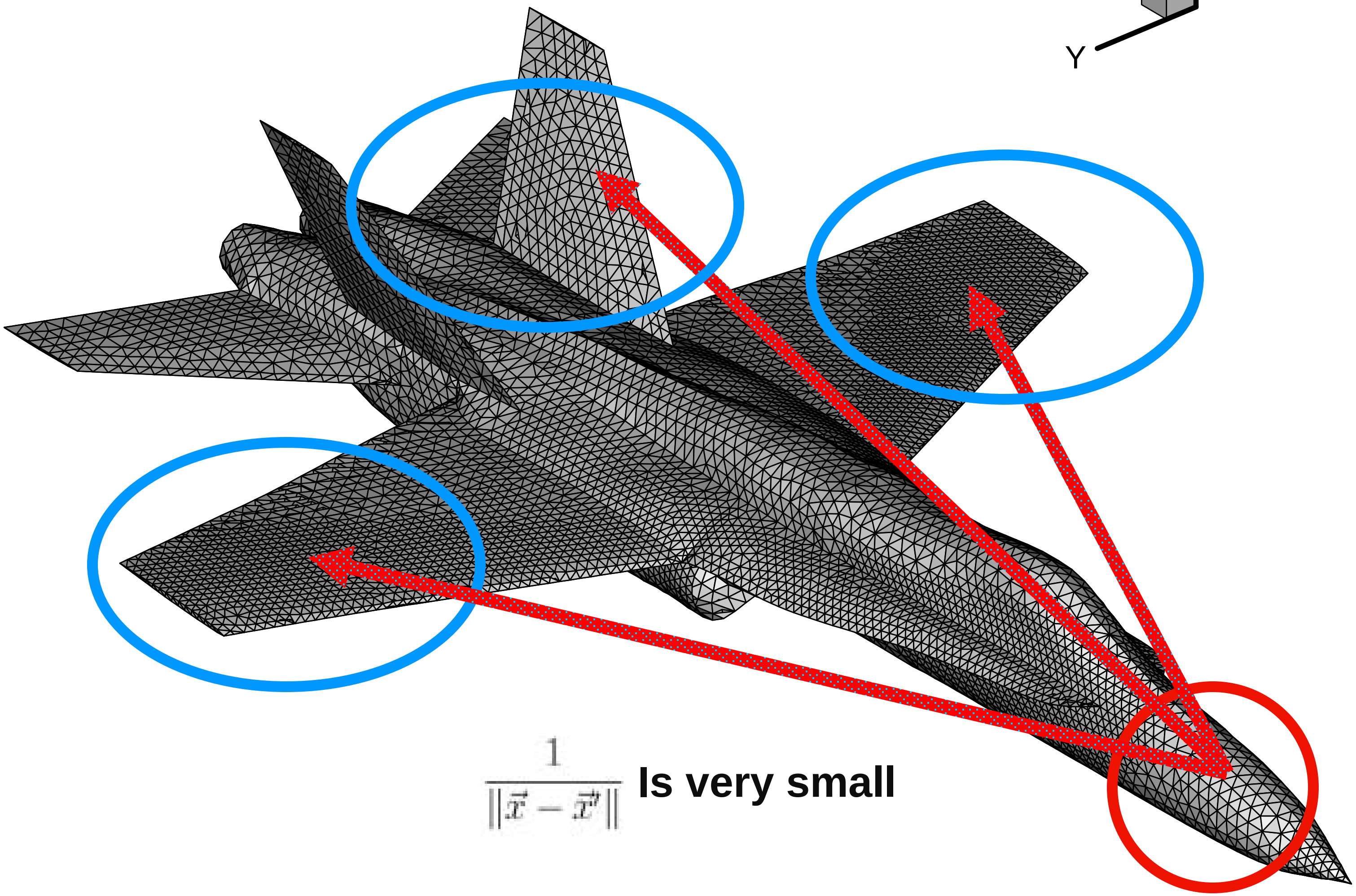
What STILL Costs So Much?

- Iterative methods:
  - Matrix vector product : Ax $(O(kn^2))$
  - *Still* computing direct interactions

- Want to approximate:
  - Matrix vector product
    - Farfield effects

Farfield Effects:





$$\frac{1}{\|\vec{x}-\vec{x}'\|}$$ **Is very small**

$$\phi(\vec{x}) = \frac{1}{4\pi}\int\int_{S_B} -\vec{V}_\infty \cdot \hat{n}_{\vec{x}} \frac{1}{\|\vec{x}-\vec{x}'\|} dS_B - \frac{1}{4\pi}\int\int_{S_{B+W}} \phi(\vec{x}) \frac{\partial}{\partial n}\left(\frac{1}{\|\vec{x}-\vec{x}'\|}\right) dS_{B+W}$$

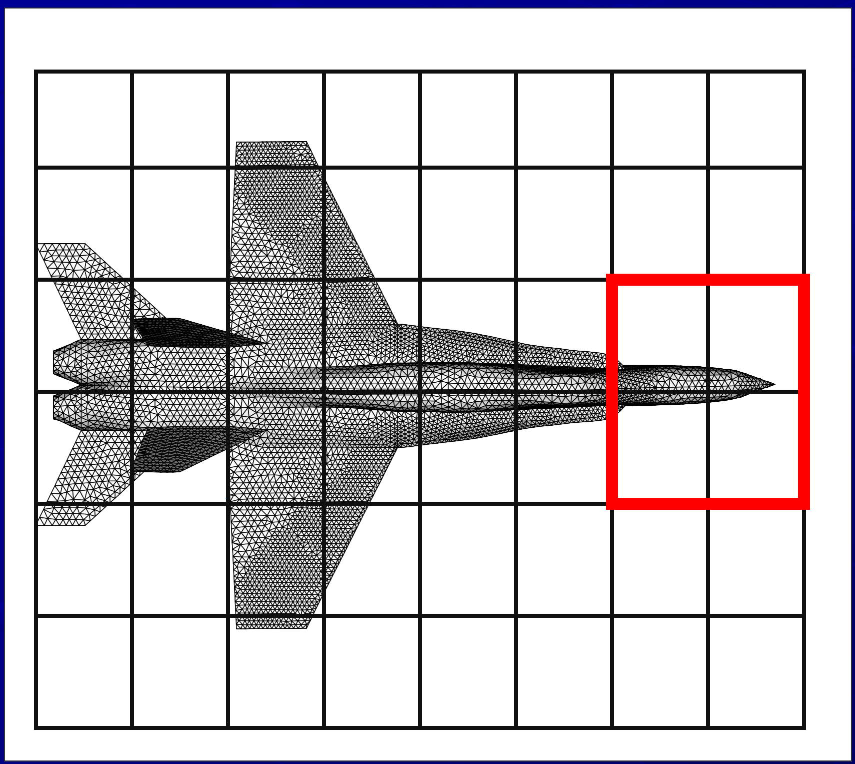
pFFT Method

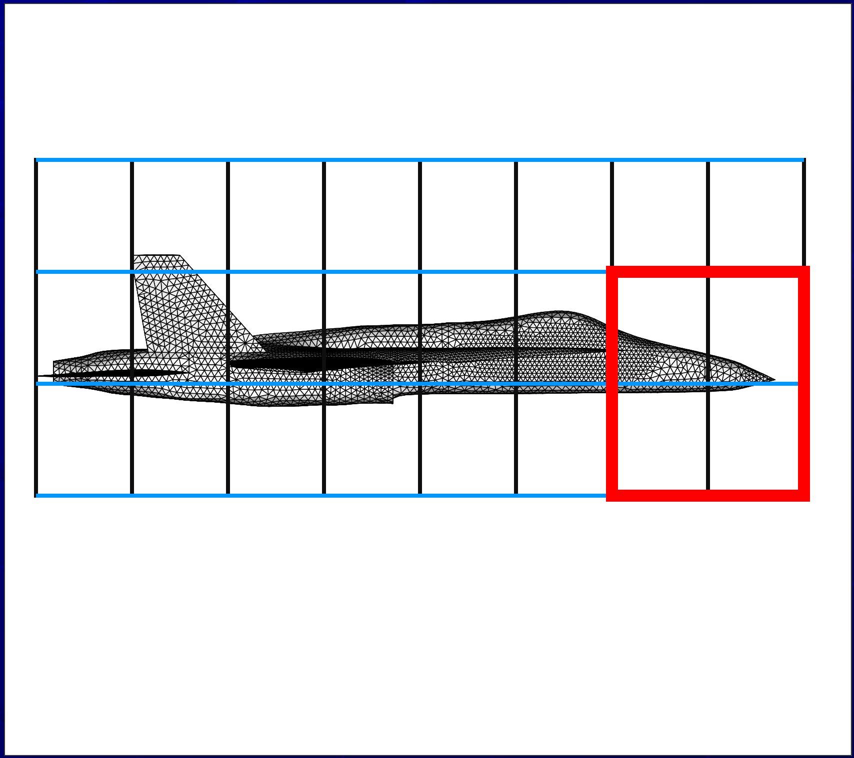
- Computes an approximate matrix vector product
  - Approximates farfield interactions
  - Directly compute nearby interactions

Phillips, J.R. & White, J.K., “A Precorrected-FFT Method for Electrostatic Analysis of Complicated 3-D Structures”

Overlay an FFT Grid

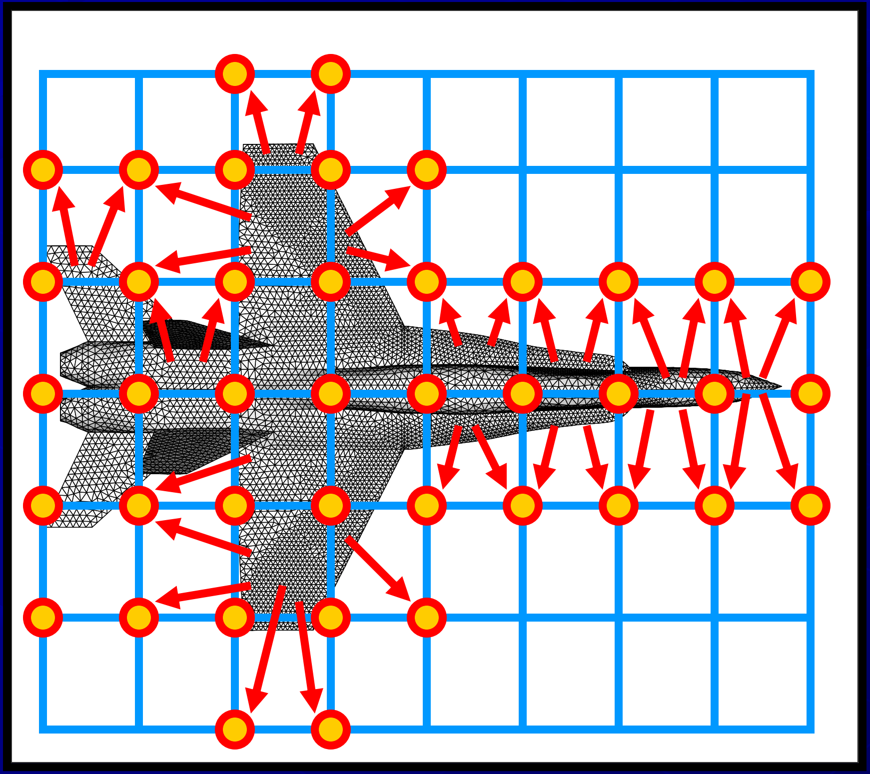
A coarse FFT grid is shown here (Nearfield Computation vs Farfield):

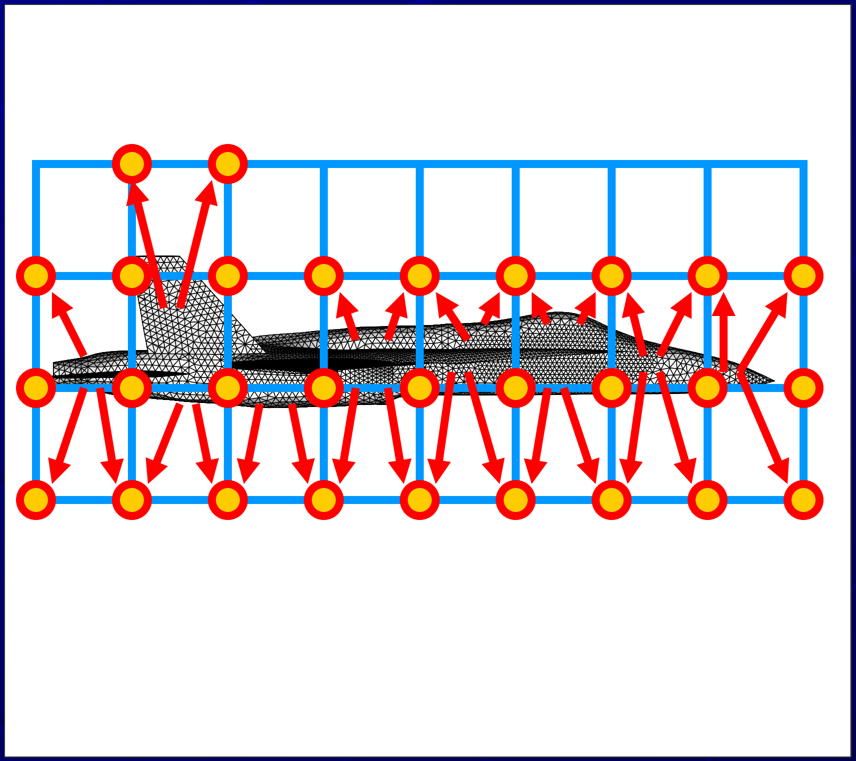




We can approximate the farfield as a 1/r computation on the grid without performing a panel integral

P = Project Panel Strength To Grid

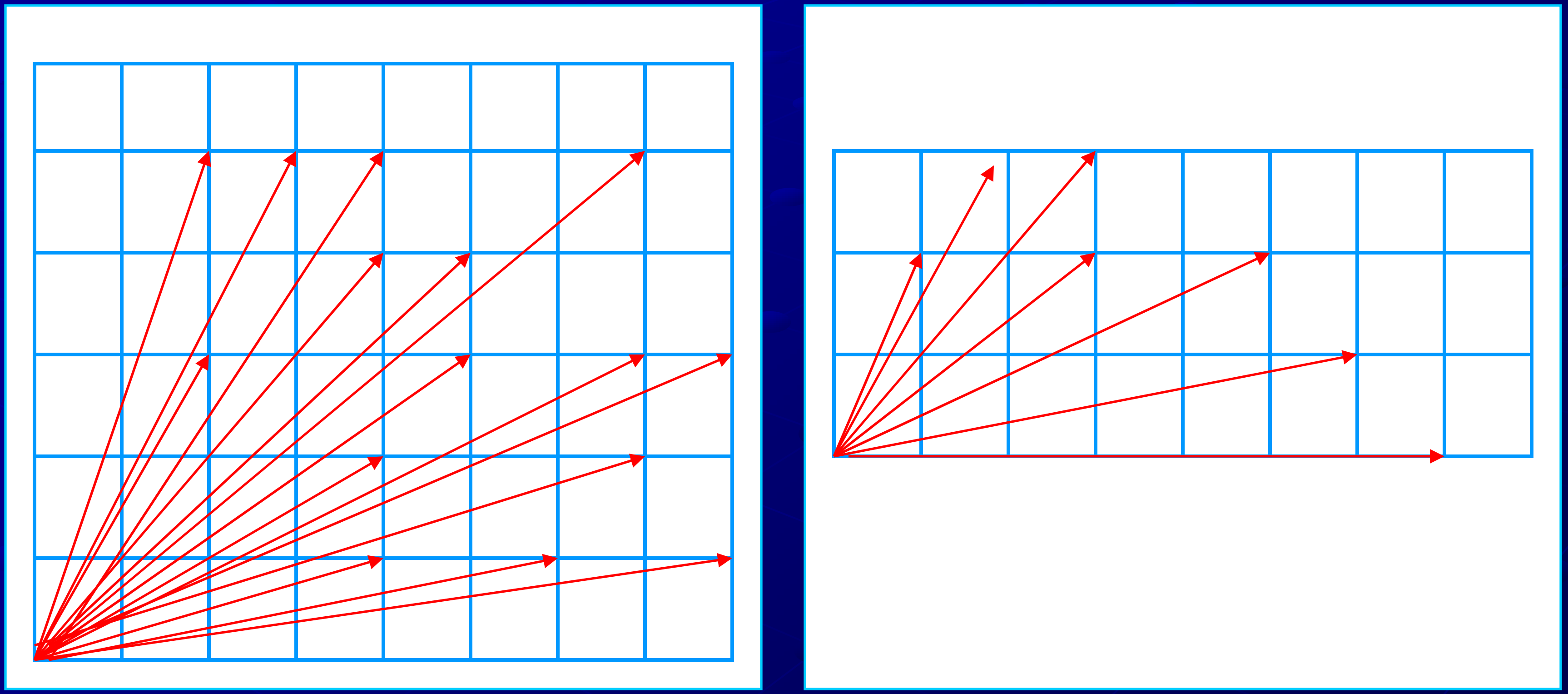




We can approximate the farfield as a 1/r computation on the grid without performing a panel integral

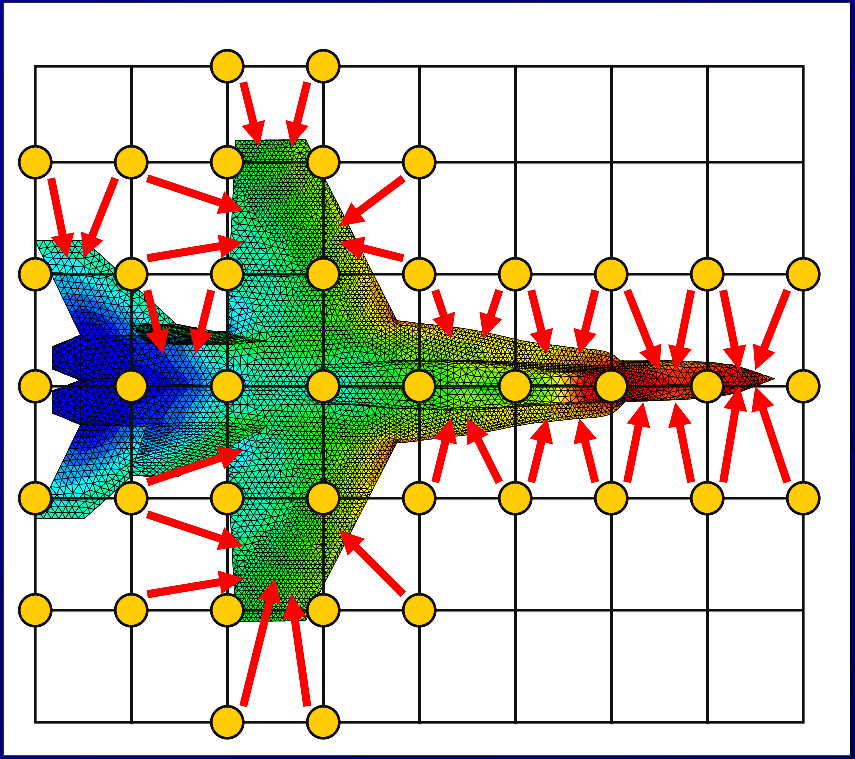
H = FFT Convolution Computation

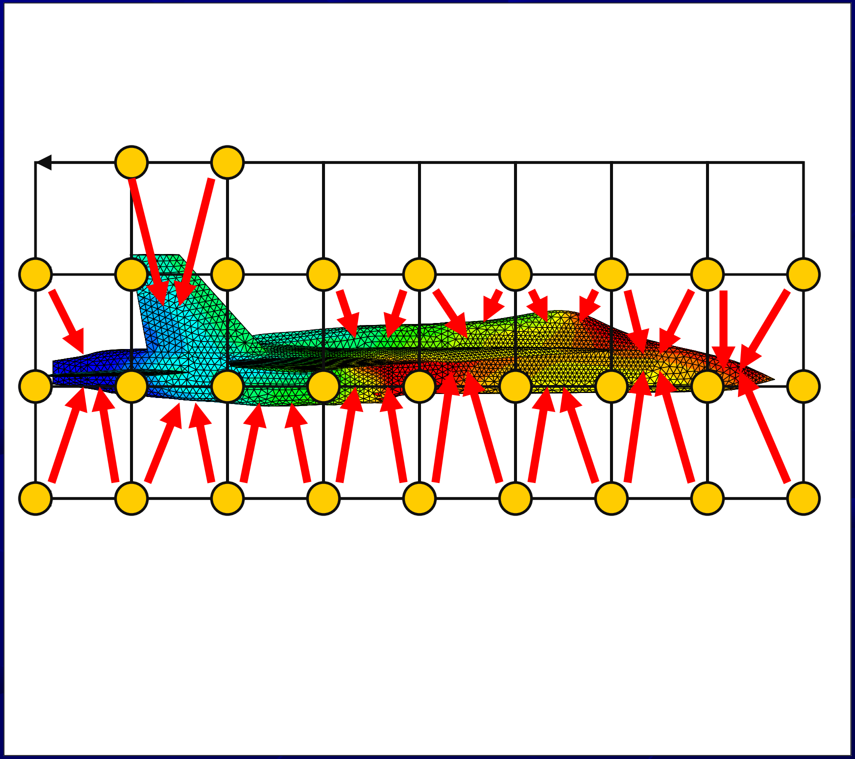
Grid Strength ➡ Grid Potential



The convolution in real space becomes a multiplication in Fourier space via an FFT

I = Interpolate Grid Potential Back to Geometry





We can approximate the farfield as a 1/r computation on the grid without performing a panel integral

Computation of Potential is:

- The grid based computation gives:

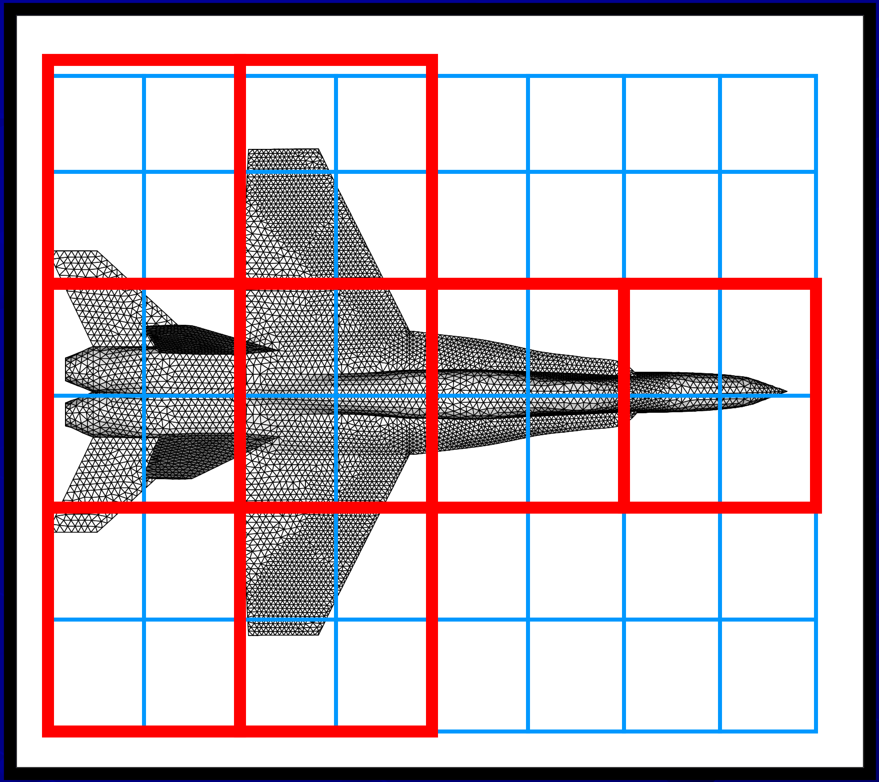
Grid Computed Potential = [I][H][P]*Source

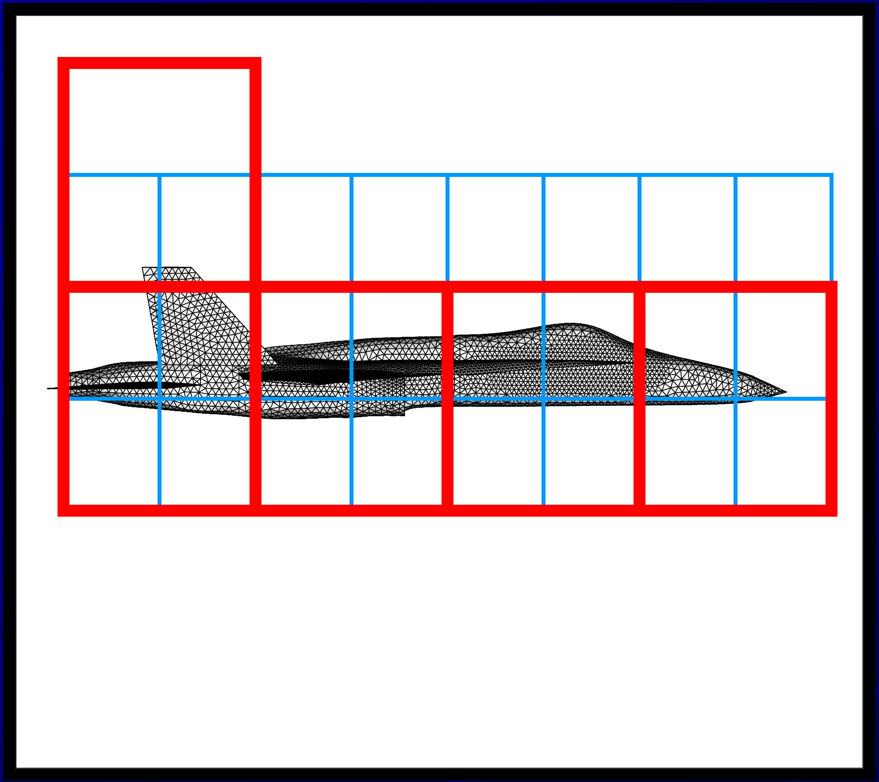
I : *Interpolation matrix*

H : *FFT*

P : *Projection matrix*

Correction : Near field Portions





$$\left[\tilde{D} - \tilde{I}\tilde{H}\tilde{P}\right]$$

The FFT Convolution effect is subtracted, and the nearfield is added

pFFT Matrix Algorithm

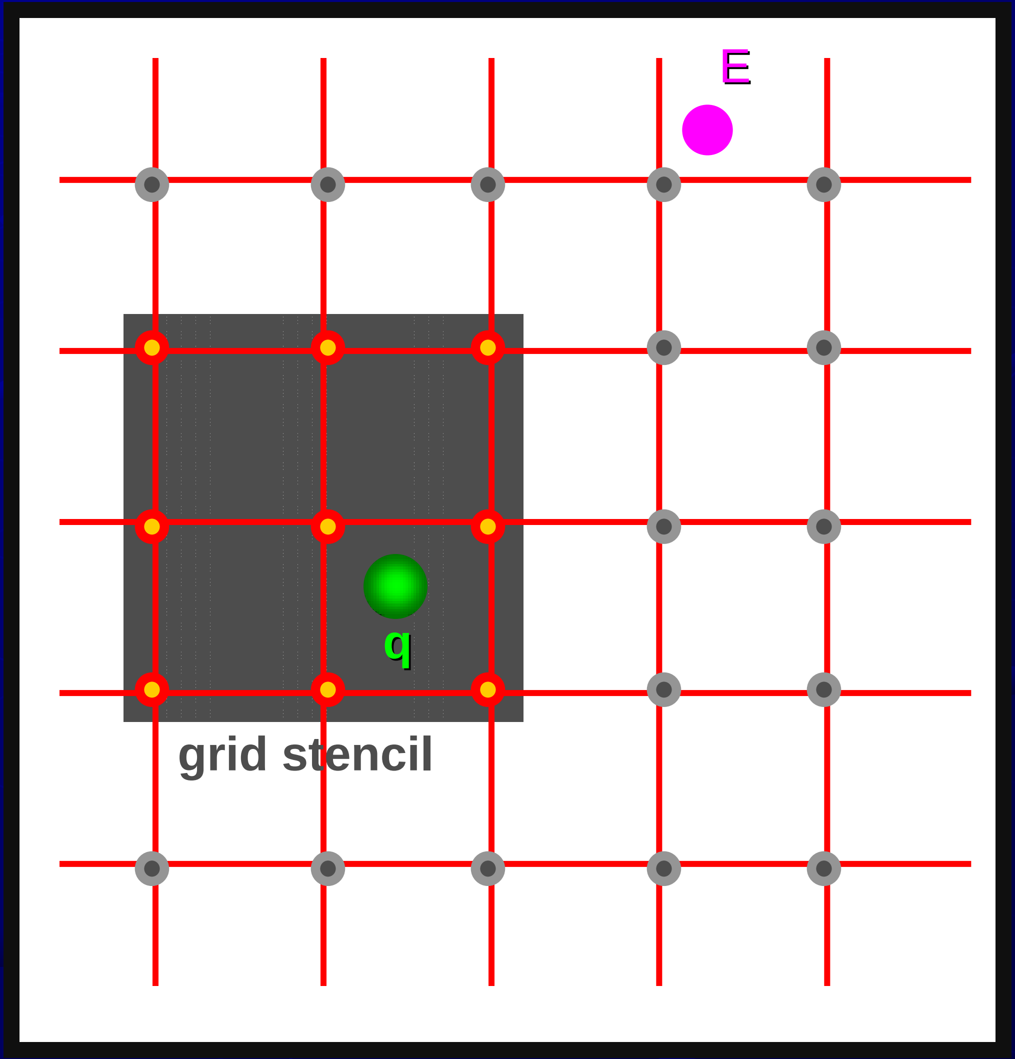
$$[AIC] \cdot \vec{\rho} = \left[IHP + \left[\tilde{D} - \tilde{I}\tilde{H}\tilde{P} \right]_{Local} \right] \cdot \vec{\rho}$$

All are sparse matrices ☺

Projection

$$\Phi_E^{(1)} = \Phi_E^{(2)}.$$

- Approach:
  - Consider a point source q in the domain.
  - Select a grid stencil
  - Construct a polynomial basis on the grid to represent grid source
  - Solve for polynomial basis coefficients by ensuring that the grid potential (2) is identical to the source potential (1) at a point E in the domain.



Projection

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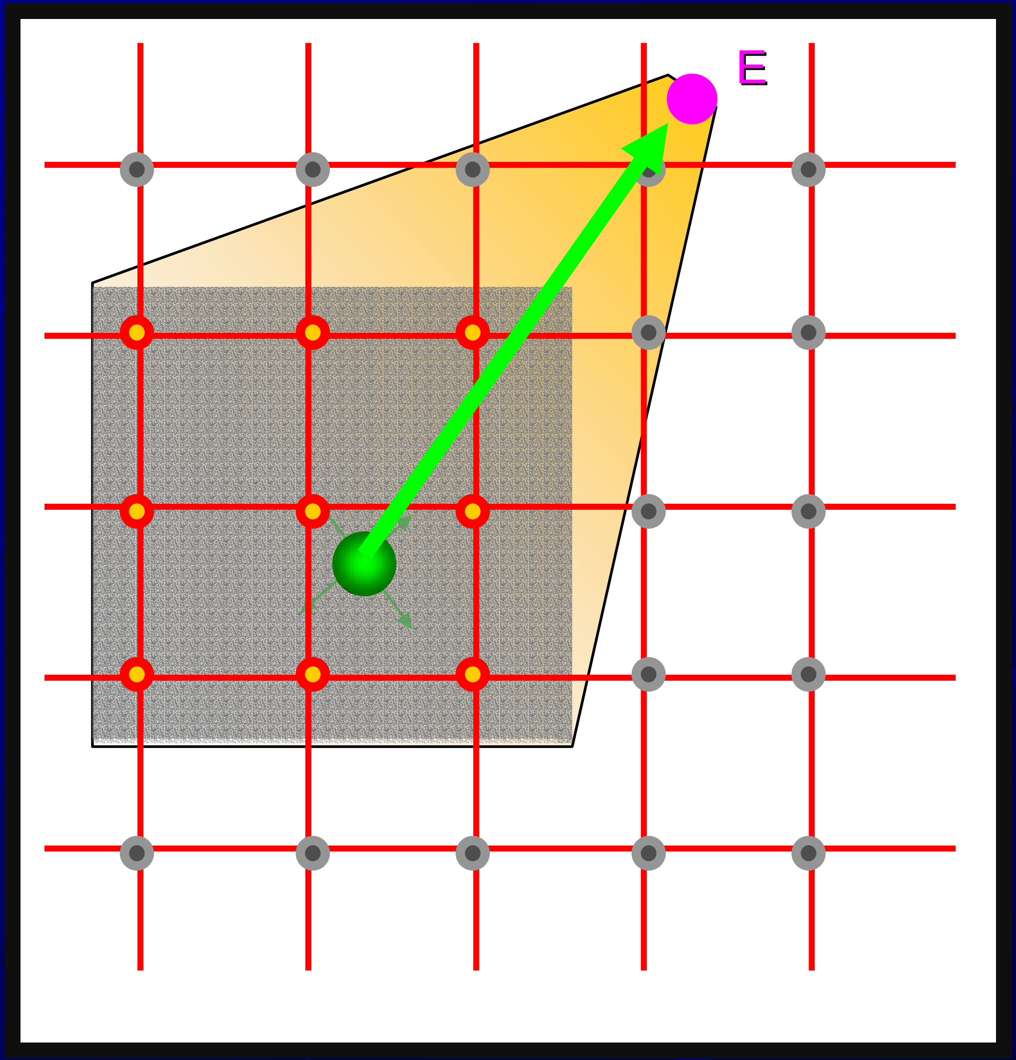
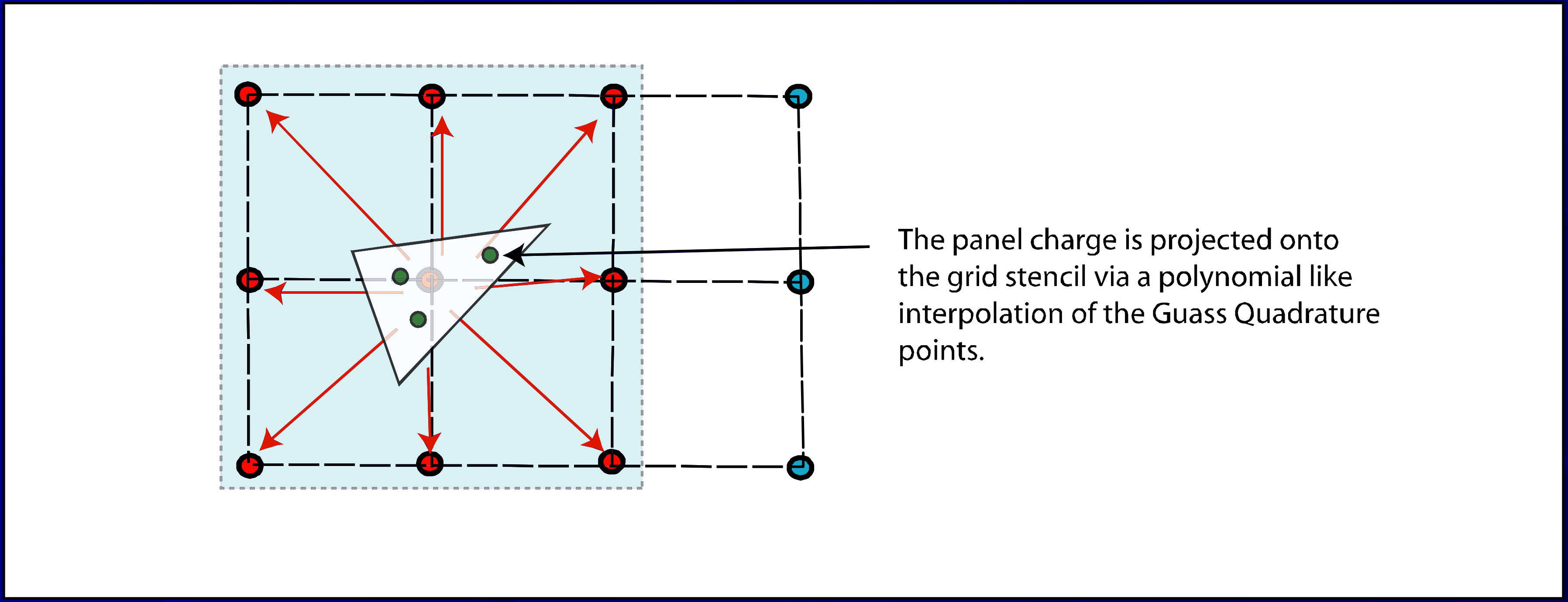
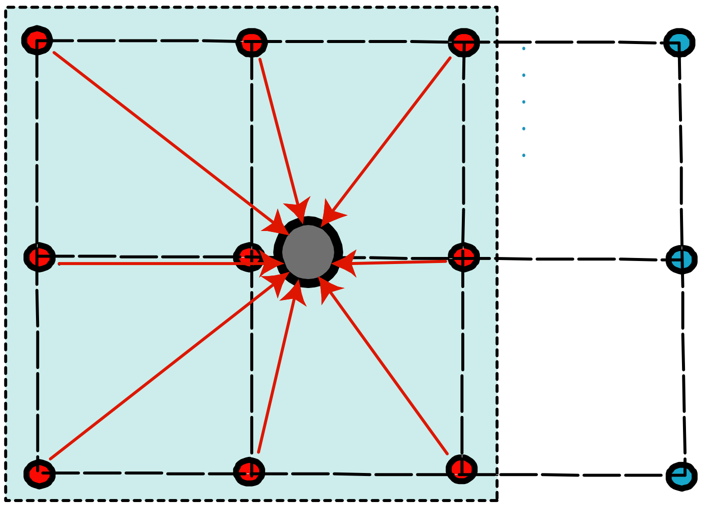


Full Panel Projection

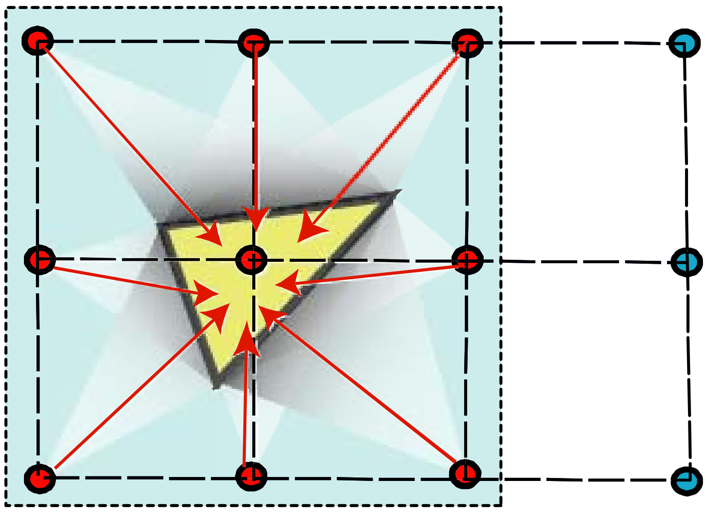


Interpolation



The grid potential is interpolated
 onto the point via a polynomial
like interpolation similar to the pro-
jection routine.

Interpolation



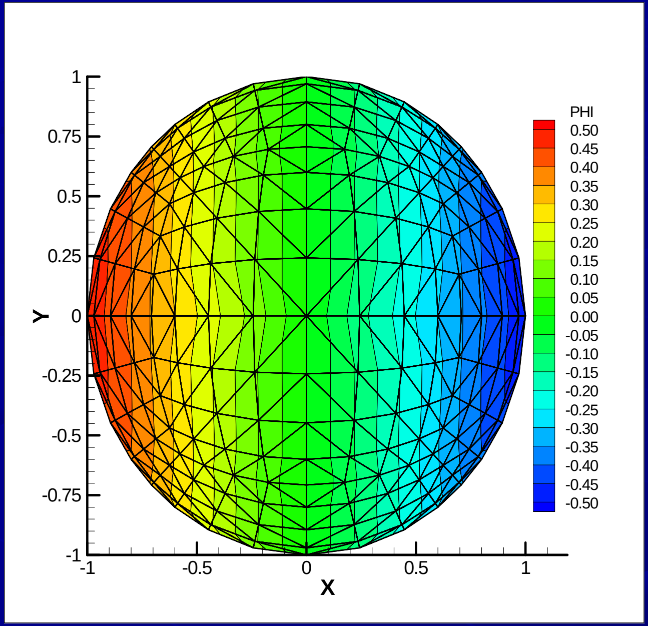
The grid potential is interpolated onto the panel via a polynomial like interpolation similar to the projection routine.

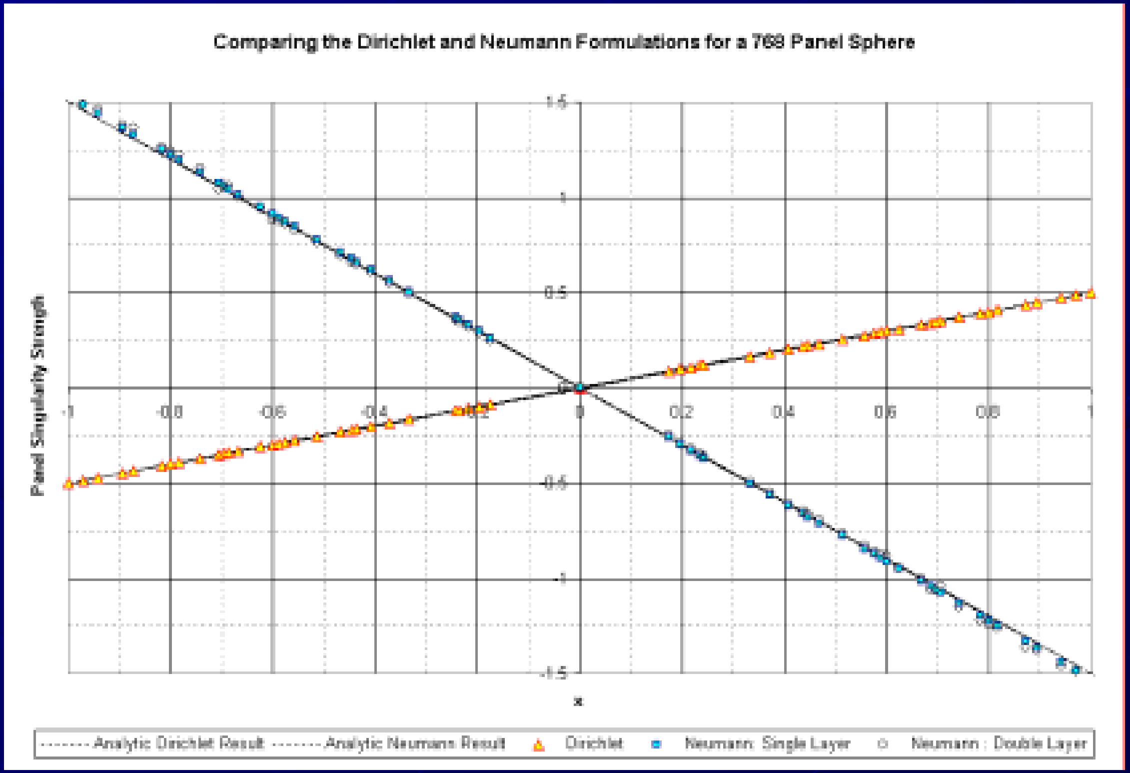
Implementation

- C++
  - Object Oriented Programming
  - Fast
- Linear Strength Panels, with higher order possible
  - Not optimized for speed
  - Not optimized for memory
- Basic tool at the moment

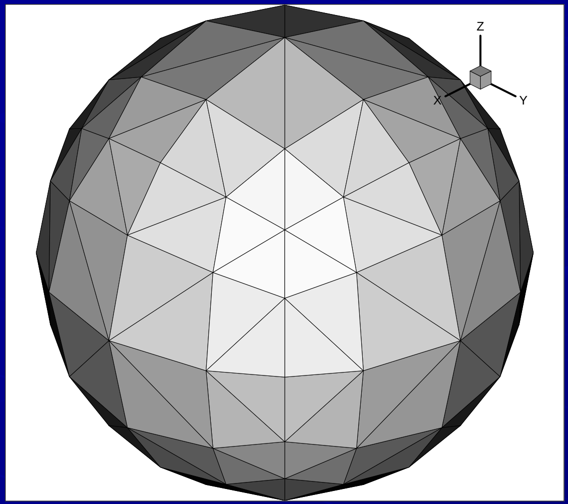
Results and Conclusions

Direct Formulation Sphere Test





Sphere Test Cases



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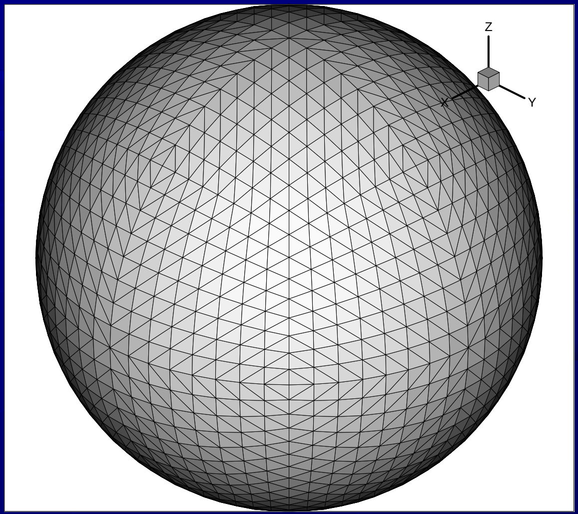

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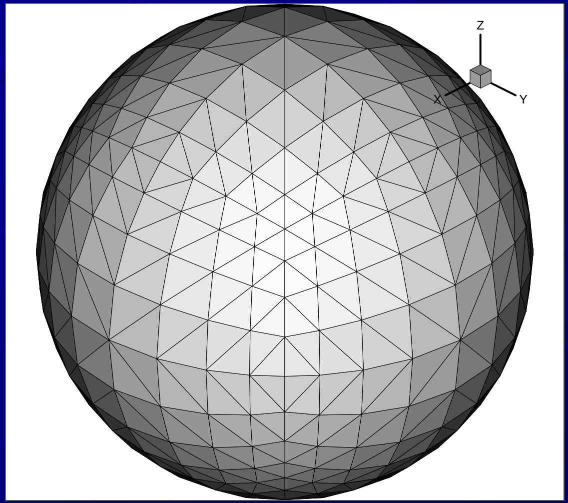

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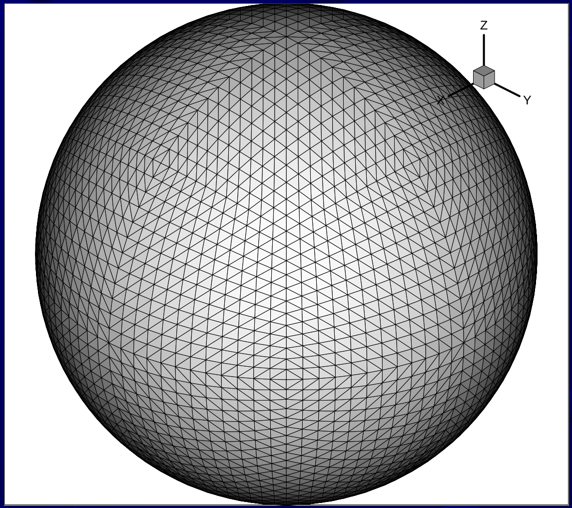
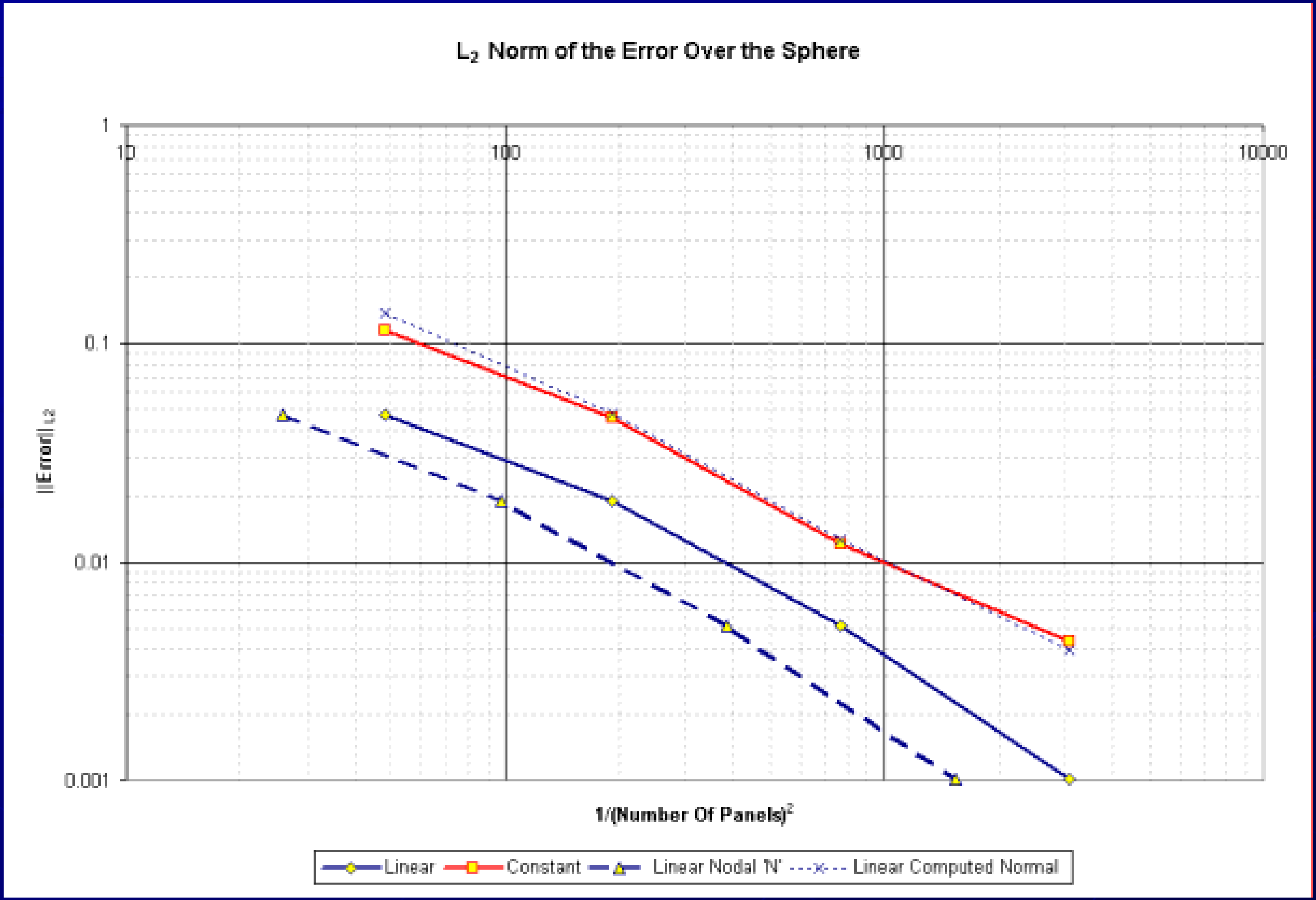
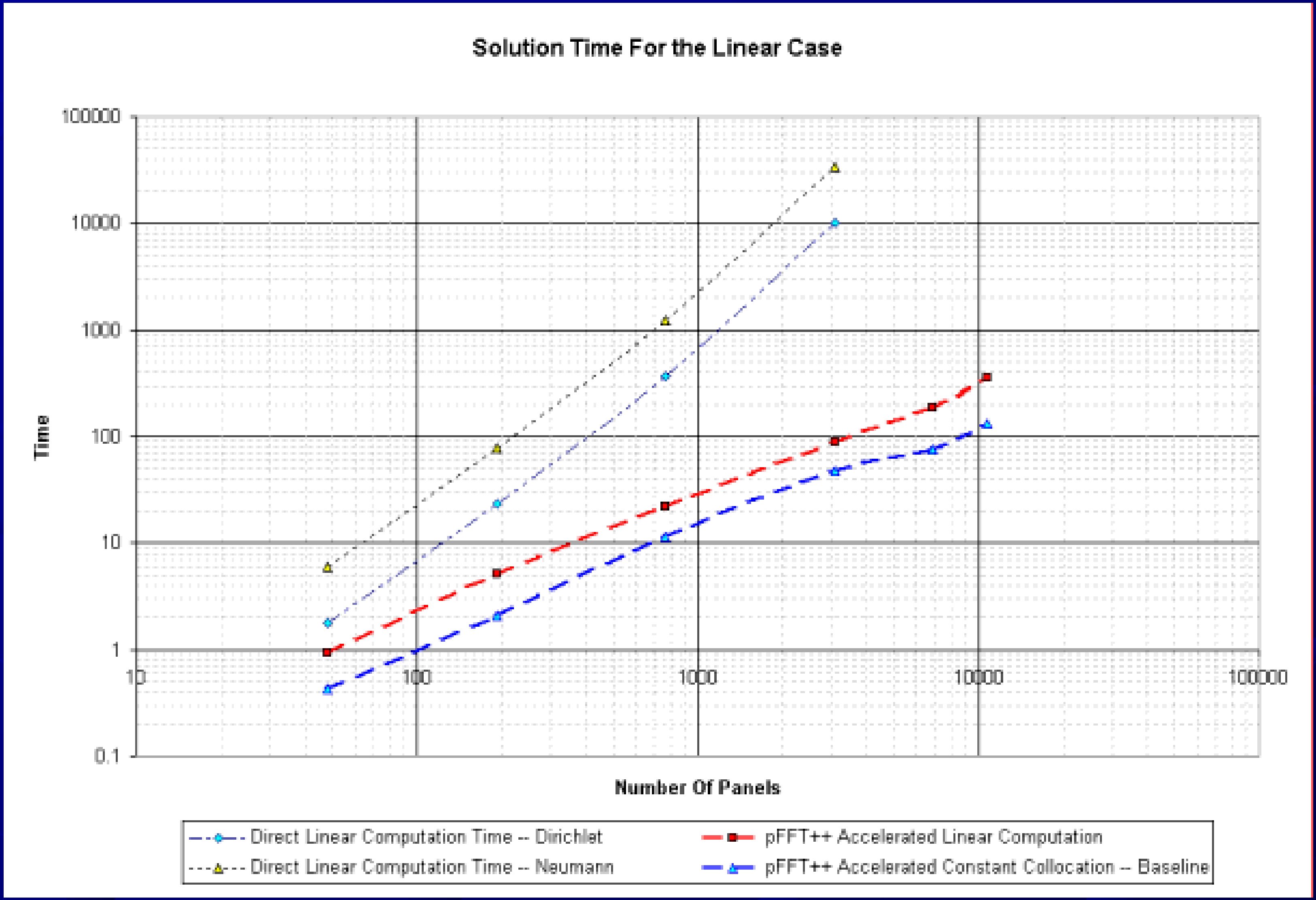


6912>

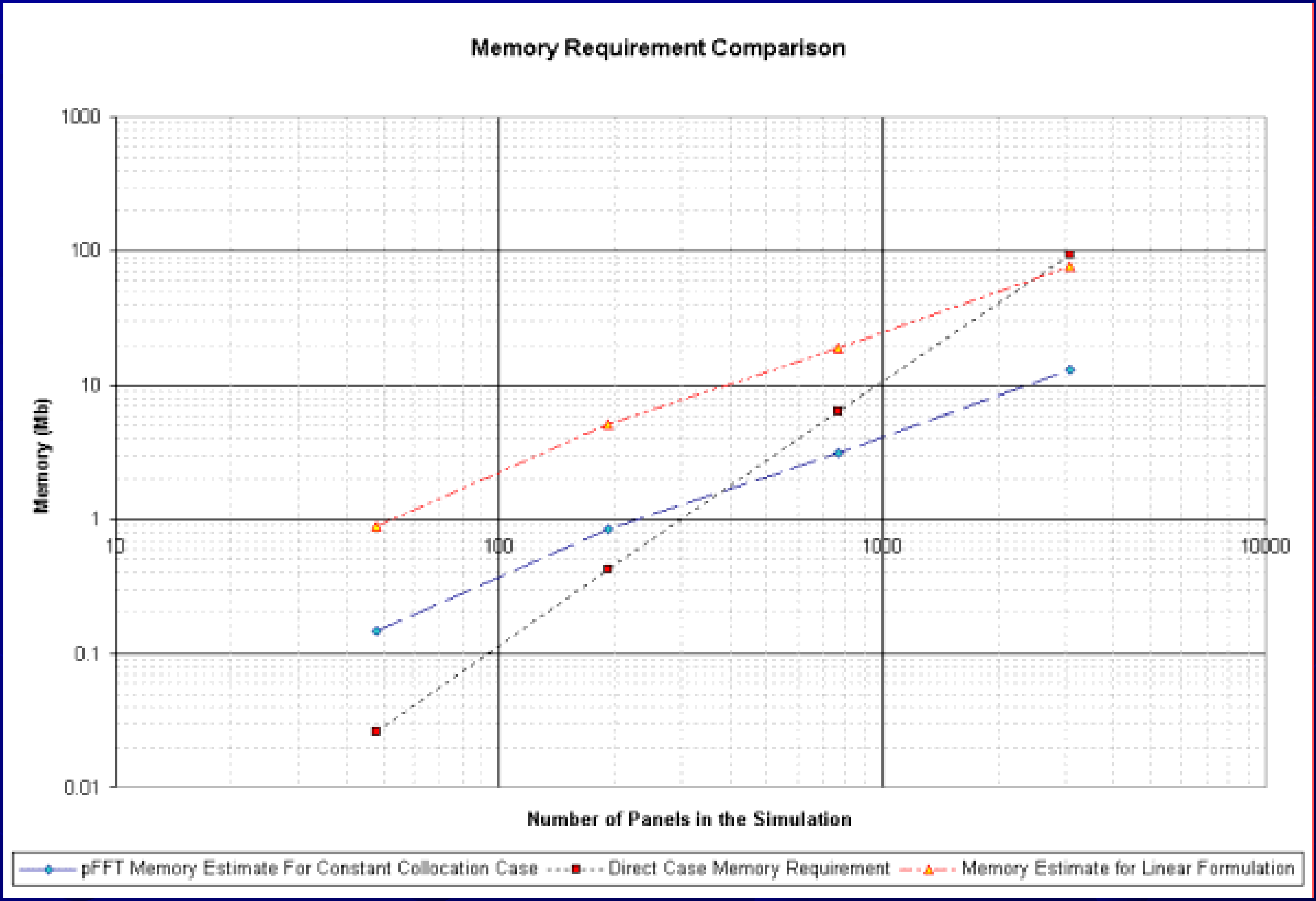
Convergence : Linear vs Constant



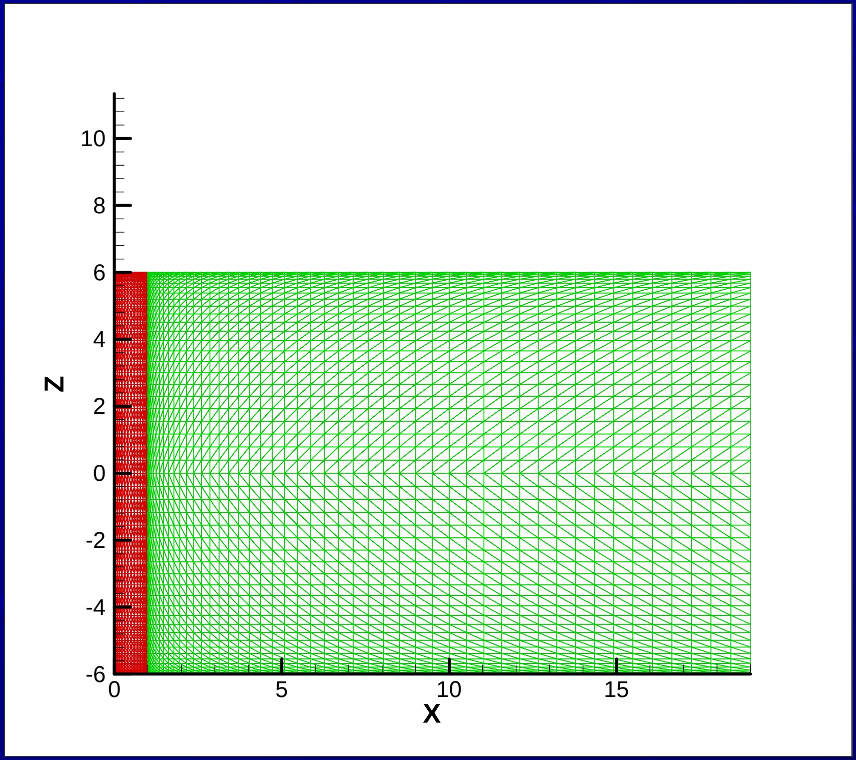
Solution Time : Linear vs Constant

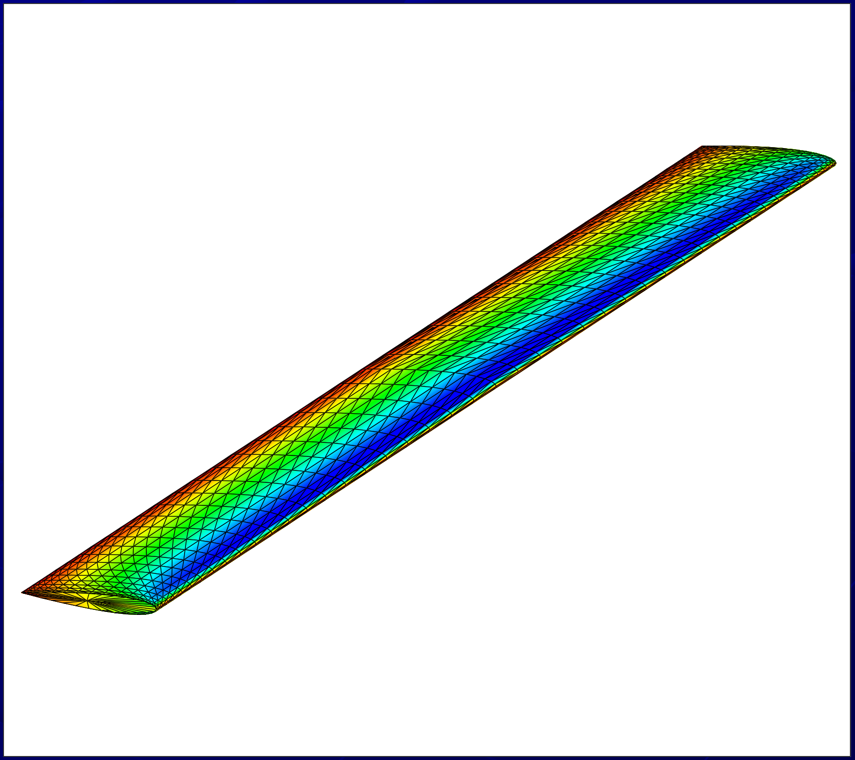


Memory – Linear Vs. Constant



Lifting Cases

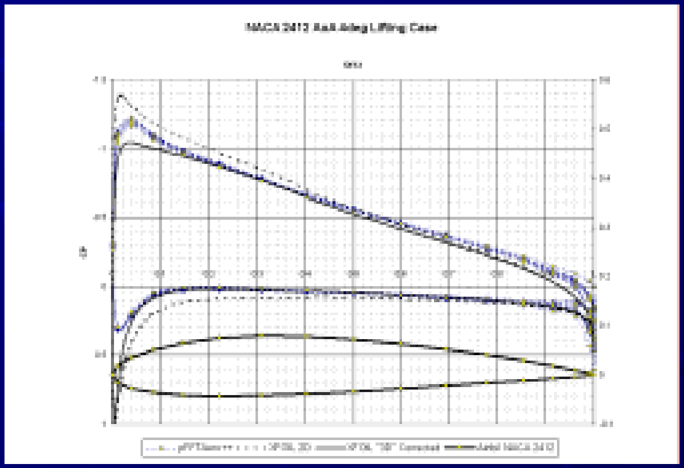


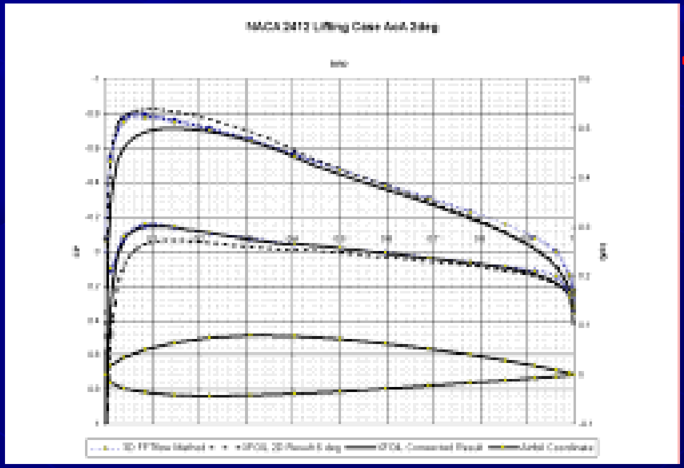


NACA 2412 -- Lifting Case

< 0 deg AoA

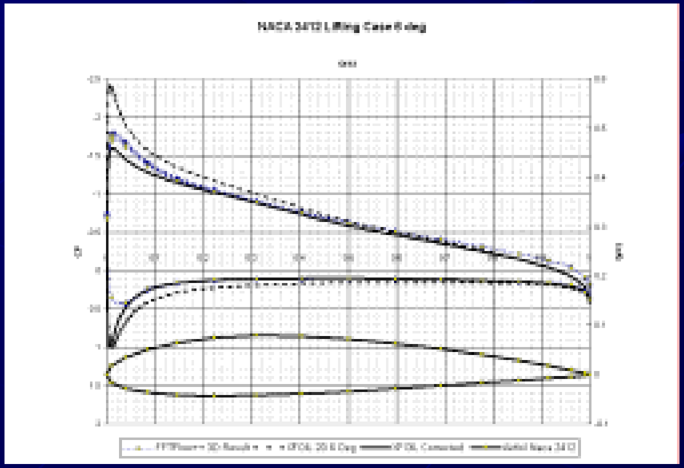
4 deg AoA >





< 2 deg AoA

6 deg AoA>



Conclusions

- Convergence
  - Limited by discretization
    - flat panels -> curved panels? – Wang et. Al.
  - Higher order extensions?
    - Quadratic
    - Cubic …
- Memory
  - pFFT++ linear implementation is not memory optimal.
    - For triangles, in optimal pFFT ++ memory is not large
- Time
  - Once again pFFT++ linear implementation is not C++ optimally coded.
    - Higher order discretization is reasonably cheap.

Summary

- Currently
  - Linear distribution of singularities on surface
  - pFFT++ and Direct solver in C++
- Future
  - Fast algorithm optimization
    - Code
    - Algorithm
  - More Post processing—what do we want to know?
    - Add on: stability, boundary layer coupling, free surface, structural coupling etc.
- PhD. Can we look at more complex fluid problems than this with this method?

Acknowledgements

- To the Singapore-MIT Alliance for their funding and support of this project.
- To my advisors: Prof. Peraire and Prof. White for their support & guidance
- To my friends & family, for their unconditional support.